A HISTORY

OF THE

Determination of the Figure of the Earth from Arc Measurements.

BY

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PREFACE.

The object of this book is to give a brief historical outline of the work done in ancient and modern times, to determine the figure of the earth from the measurement of meridian arcs. The results here gathered are intended to show the progress and development of the work, thus enabling one to obtain a comprehensive idea of what has been accomplished in the subject, and to note the progress in methods and the precision attained.

The results of the early investigators are given at full length, for ordinarily their reports are not accessible. Those of later investigators, the results of whom can be found in most libraries, are more condensed. Of the theory of the figure of the earth and methods of determination of its form from given data, only so much is given as is essential to the discussions of this paper.

Few direct references have been made, but the subject matter has been obtained from the original reports in nearly all cases since the time of Norwood. At the close of the book there is given a list of references and works consulted. Where allusions in the text are made to the reference books, they occur in the following form (12, p. 92), meaning reference book numbered 12, and page 92 of that work.

ARTHUR D. BUTTERFIELD.

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INDEX.

A HISTORY OF THE DETERMINATION OF THE FIGURE OF THE EARTH FROM ARC MEASUREMENTS.*

ARTHUR D. BUTTERFIELD, M. S.

Our oldest knowledge of civilization goes back to the Chaldeans of Mesopotamia. From these people came the first impulse of practical thought and of scientific attainment. They were primarily a shepherd people, and as such must have seen the heavens night after night dotted with constellations, the positions of which varied according to the times and seasons. Gradually different facts must have been observed and an explanation sought, since from the beginning there have been those in all nations to whom phenomena have appealed, and inquiring, thoughtful minds have been led to seek the reasons for those phenomena. In this manner from simple observation the science of astronomy had its rise.

Some facts of their knowledge, transmitted as a heritage to subsequent nations, have come down to us. History ascribes to them the invention of the sexagesimal system of angle measure, developed doubtless from knowing approximately the length of the year; also, the grouping of constellations; some knowledge of comets; and recognition of the motion of the moon's apogee from the observed recurrence of lunar eclipses in cycles of eighteen years. Much of their civilization was undoubtedly lost when they as a nation decayed and disappeared, but the facts we do possess tend to show that astronomy was the most highly developed science of very ancient times.

Practically all of the astronomy of antiquity was purely observational, having to do with the rising and setting of the sun, moon and stars; and while people were only concerned with these, the actual figure of the earth was not necessary to them. Its form and dimensions would be sought for, as adding to the general knowledge. When, however, astronomy began to seek to interpret the motion, distances and location of other bodies, then the figure of the earth became absolutely indispensable, for it was the unit upon which everything else depended. The real awakening as to its value did not take place until Sir Isaac Newton announced the law of attraction in his Principia, published in 1686. Thus while the question of the figure of the earth is an old one, actual scientific investigations have been made only in comparatively modern times, and the most precise work has been done in the last few decades.

EARLY CONCEPTION OF THE FIGURE OF THE EARTH.

What the Chaldean conception of the figure of the earth was we are not absolutely sure, but from their knowledge of astronomy we infer that they recognized its spherical shape. Gore, in his Elements of Geodesy (12, p. 1,) makes this statement: "Froriep refers to a Sanscrit manuscript containing the following sentence: 'According to the Chaldeans, 4000 steps of a camel made a mile, 66⅔ miles a degree, from which the circumference of the earth is 24,000 miles,'" Our first authentic knowledge of the earth's form goes back to the time of the early Greeks, and with them there was a diversity of opinion.

Anaximenes, (abt.) 500 B. C., held the view that the earth was an extended flat surface, and this was the common opinion.

Anaximander, 611-545 B. C., believed it to be cylindrical in shape, in the form of a drum, with the sea and land on the top surface.

Plato, 429-348 B. C., believed it was a cube.

Some few, particularly the members of the Pythagorean School, believed in the spherical form, but this was generally rejected.

Aristarchus, 310-250 B. C., an astronomer, deserves special mention, for he taught that the earth moved in an orbit around the sun, and at the same time revolved on its own axis. He also determined approximately the relative distances of the sun and moon from correct astronomical principles.

ERATOSTHENES, 275-194 B. C.

To Eratosthenes, astronomer and librarian of the University of Alexandria, belongs the honor of the first arc measurement for determining the size of the earth. The measurement was made about 250 B. C., and the method employed was correct in principle, and has been used to the present day without any changes. The principle is to compare a meridian arc on the earth with a like arc in the heavens. It amounts to finding, 1st, the meridional distance between two points on the earth's surface either by direct measurement or computed measurement from a base line and triangulation; 2nd, the latitude of the two places from observations on the stars. The difference in latitude gives the arc in the heavens; therefore knowing this and the terrestrial distance, the value of the unit arc on the earth's surface, usually taken as one degree, is found. Modern measurements differ only in the care and methods taken to obtain precise results, for naturally the first measurements were crude, and some assumptions were made.

According to Cleomedes, Eratosthenes noticed at Syene (now Assouan), in Upper Egypt, that the sun at noon of the summer solstice was directly vertical, the observation being made by noticing that the edges of a deep well cast no shadow on the bottom. It may be questioned whether Eratosthenes himself made the observation at Syene; it is more probable that he used the fact, based on the observations of others.

At Alexandria, at the same season, he found by the use of a hemispherical bowl with a vertical style casting a shadow, that the sun's zenith distance was $\frac{1}{50}$ of a great circle, or $7\frac{1}{5}$ degrees. He assumed Syene and Alexandria to be in the same meridian—in reality they differ about two degrees in longitude—and estimated the distance between them to be 5000 stadia. From this he obtained the circumference of the earth to be 250,000 stadia. This later was increased to 252,000 stadia in order to have a round number, 700 stadia, as the value of one degree.

POSIDONIUS, 135–45 B. C.

There were two men of antiquity of that name who were more or less famous, and there seems to be a disagreement as to which of the two made the measurement, and also as to where it was made.

The first Posidonius (14, p. 1123) was a mathematician and astronomer at Alexandria about 200 B. C.; the second, (3, Vol. 3, p. 506) was a Stoic philosopher of Rhodes, 135-45 B. C., who traveled extensively and finally settled in Rome. Cleomedes asserts that at Rhodes he observed that the star Canopus was just on the horizon, while at Alexandria its meridian altitude was estimated to be $\frac{1}{48}$ of a great circle, or $7\frac{1}{2}$ degrees. The distance between the two places was assumed to be 5000 stadia, thus giving 240,000 stadia as the circumference of the earth.

Strabo (3, Vol. 3, p. 506) asserts that the observation was not made at Rhodes, but in Spain, and that the result was 180,000 stadia as the circumference.

Bailly (1, Vol. 1, p. 166) has ascribed to Posidonius a second arc measurement, which is that the difference of observed zenith distances of the sun, at Syene and Lismachie, was $\frac{1}{15}$ of a great circle, and that the estimated distance between the two places was 20,000 stadia, thus giving for the earth's circumference 300,000 stadia.

If we ascribe the measurement to the first Posidonius, it means that these measurements were made during the life of Eratosthenes, and it is quite certain that they were made afterwards; this view would be strengthened if there were a second measurement, as that would seem to indicate that the investigator wished to exceed Eratosthenes' work and reputation. Hence we may conclude that they are due to Posidonius, the philosopher. Again, as to where made; if made in Spain, the place compared with it could hardly have been Alexandria, for this would have been a very oblique arc, and they, at that time, were hardly prepared to discuss such; and since Rhodes was his home, it is more natural to infer that the observation was made there. Regarding the second alleged measurement, it means comparing with Syene (now Assouan) a point in latitude 48 N. (approximately through Germany or Southern Russia), or a point in Africa on the equator. Accurate geographical knowledge of these places is doubtful, and the measurement may be questioned.

The result of the observation at Rhodes and Alexandria was probably not as accurate as that of Eratosthenes, beacuse of the error brought in by refraction, this being a maximum when on the horizon, and also from the impossibility of estimating accurately the distance of the two places separated by the Mediterranean Sea. How near these early crude measurements came to the truth can only be conjectured; the value of their units of measure are not known, with anything like precision, in terms of our modern standards. If we should take the Olympic stadium, which is supposed to be 202¼ English yards, the circumference of the earth, according to Eratosthenes, would be about 30,000 miles.

Naturally we should, from the wide range of values, class the results as very poor, for we have as the earth's circumference from

Eratosthenes	(about 225 B. C.),	astronomer,	250,000 stadia
Posidonius	(about 75 B. C.),	philosopher,	240,000 "
			300,000 "
Cleomedes	(probably 200 A. D.),	astronomer,	300,000 "
Ptolomy	(about 150 A. D.),	astronomer,	180,000 "
Strabo	(about 25 B. C.),	geographer,	180,000 "

From the above, we can assert that little reliance can be placed upon the results, or that the values of the stadium were different. Bailly (1, Vol. 1, pp. 143-168) has devoted considerable space to showing that these measurements are all practically the same, the units, in the various places, bearing the same relations to each other, that the given circumferences bear to one another. In this way, and by making other various assumptions, and working backwards from our present day knowledge, some writers have brought about a reasonably close agreement between the early measures and the truth. Since, however, we cannot speak with accuracy as to their results, we can, nevertheless, honor them for starting the work and setting such a worthy example of investigation.

AL MAMUN. 814 A. D.

A period of about one thousand years now elapsed before any new measurements were made. During the rise of learning among the Arabs in the ninth century, Caliph Al Mamun ordered a measurement made, about the year 827 A. D. Two parties started from the same point, on the Mesopotamia plain of Sinjar; one measured toward the north, the other toward the south, and in as straight a line as possible. Each party measured until they had changed their latitude one degree. They then met and compared distances; one party made a degree equal to 56 miles of 4000 coudees each, the other 56⅔ miles of 4000 coudees. The latter value was the one adopted, but the reason for it we do not know, neither do we know which party made the greater measurement, nor the length of the coudee, which is variously given as the length of 24, 27, or 32 doigts, each doigt being a length equal to 24, 27, or 32 kernels of a certain kind of grain placed side by side (1, Vol. 1, p. 56).

FERNEL.

Another long period, about 700 years, elapsed before the next measurement took place. It was made by Fernel near Paris in 1528, the result being published by him, in the same year. The measurement was made by the revolution of his carriage wheel, in driving from Paris to Amiens. He observed the sun's meridian altitude, at Paris, on August 25, 1528, then went north, as nearly as he could judge, until he found his latitude had been changed one degree, taking into account the change in declination of the sun.

The meridian altitudes were measured by using a triangle two of whose sides were eight feet in length; one of these sides was held vertical, the other, pivoted to the first, was movable and carried sights. The third side was used to measure the distance between the other two, and was graduated to single minutes, thus serving as a chord of a sector. The result obtained for one degree was 57,746 toises, 69.12 miles, according to Delambre. De Morgan (19, Vol. 19, 1841, pp. 445-447), has taken exception to this, for Fernel gave his results as 68.096 Italian miles, and as the Italian mile was supposed to be 1628 yards, the results would be about 63 miles. As many of their standards depended on the assumed length of a pace, it is impossible to assign strict values to their results.

SNELL.

In 1617, Willebrord Snell published, in Leyden, the results of his work. In his operations, he made a great advance over all predecessors, in that he introduced trigonometric methods, so that instead of measuring the whole distance he measured a short base, and by means of a system of triangles, computed the distance between the two extreme places.

Near Leyden, he measured a base of 326.43 Rhinland perches, a perch being 12 Rhinland feet. Working from this, he established geometrically the relation between the principal cities of Holland and also several in Flanders, extending his work as far north as Alcmaer and as far south as Bergen-op-Zoom. The latitude of these three places was observed from stars. The difference in latitude between Alcmaer and Bergen-op-Zoom, was 1° 11′ ½, and the meridian distance corresponding to the latitude stations was 33,930 perches, giving as a value of 1 degree, 28,473 perches. By using Alcmaer and Leyden, the length of one degree was found to be 28,510 perches; the round number 28,500 perches was selected as the probable value. As the relation of the Rhinland foot to the French foot was accurately known, it was possible to express the result in the French standard. Using the relation given in the following table, the length of one degree would be 55,100 toises.

In observing the angles, of the triangles, a quadrant of a circle, made of copper and about two feet in radius, was used; this was graduated to two minutes, and in reading, estimated to single minutes. The astronomical observations were made with a like quadrant, but of 5 ½ foot radius.

7 ? Cassini, in 1~~6~~07, (24, 1718, p. 289), observed the latitude at several of Snell's stations and was led to believe that the results given were small, and that there were small errors in both the triangulation and latitude observations. Re-observations of latitude and recalculations, by Muschenbroek, in 1729, gave as the value of 1 degree, 57,033 toises.

NORWOOD.

During the years 1633-1635, Richard Norwood, an Englishman, made an arc measurement between London and York. The results according to the author's preface, were written up in 1636 and published under the title of The Sea-man's Practice, of which there were several editions. His method of work is shown by the following quotations taken from the edition of 1689. (18, pp. 4-5.)

"Upon the 11th of June, 1635, I made an Observation near the middle of the City of York, of the meridian altitude of the Sun, by an Arch of a Sextant of more than 5 foot Semi-diameter, and found the apparent Altitude of the Sun that Day at Noon to be 59 deg. 33 min.

I had also formerly upon the 11th of June Anno 1633, observed in the City of London, near the Tower, the apparent

Meridian Altitude of the Sun, and found the same to be 62 deg. 1 min. And seeing the Sun's Declination upon the 11th day of June, 1635, and upon the 6th day of June, 1633, was one and the same, without any sensible difference; and because these Altitudes differ but little, we shall not need to make any alteration or allowance, in respect of Declination, Refraction or Parallax.

Wherefore subtracting the lesser apparent Altitude, namely 59 deg. 33 min. from the greater 62 deg. 1 min. there remains 2 deg. 28 min. which is the difference of Latitude of these two Cities, namely of London and York." - -

"Coming at that time from thence to London I further found by measure, that the parallel of York is from the parallel of London 9,149 chains: every Chain being 6 Poles and every Pole 16½ of an English Feet. That is, every chain is 99 feet." - - - - - - - - -

"Yet having made Observations at York, as aforesaid, I measured (for the most part) the way from thence to London; and where I measured not I paced, (wherein through Custom, I usually come very near the truth) observing all the way as I came with a Circumferentor all the principal Angles of Positions for Windings of the Way (with convenient allowance for other lesser windings, Ascents and Descents) and these I laid not down by a protractor after the usual manner, but framed a Table much more exact and fit for this purpose as we Shall after Shew: so that I may affirm the Experiment to be near the Truth."

From this measurement Norwood found the length of 1 degree to be 367,196 feet, or 367,200 feet, as he called it, in round numbers. From the quotation, it is noticed that he does not say whether he used the same instrument for observing at London as at York, but he seems to lay particular stress on the fact that the observations were made on the same day of the month, and that the sun's declination was practically the same.

At the time of these early arc measurements, there was no such thing as uniformity regarding standards of length; each country or state or even a city had its own standard. One result of arc measurements, and a very important one, has been the bringing about of an universal standard, that of the metre. Because of this diversity of standards, it is often difficult to express the value of the measurement in one standard, in terms of those of another, with anything like precision. Approximate relations were known between the different standards, and these are perhaps as accurate as the measurements themselves.

The two following tables are given as showing the considered relations, at the time they were published.

Table given by Richard Norwood, in the Sea-man's Practice, (18, pp. 33–34) showing the relation between the English foot, and those of other countries, about the year 1650.

Ancient Roman	foot –	– equals –	1.033	Eng.	ft.
" Greek	"	"	1.076	"	"
" Babylonian	"	"	1.211	"	"
" Alexandrian	"	"	1.240	"	"
" Antiochian	"	"	1.405	"	"
" Arabian	"	"	1.102	"	"
Rhynland	"	"	1.033	"	"
Dort	"	"	1.085	"	"
Middleburg	"	"	.992	"	"
Amsterdam	"	"	.934	"	"
Antwerp	"	"	.939	"	"
Lavian	"	"	.939	"	"
Hafnian foot in Denmark		"	.965	"	"
Paris foot called King's foot		"	1.090	"	"

Venice	foot - -	equals -	1.157	Eng.	ft.
Toledo	"	"	.896	"	"
Noremberg	"	"	1.006	"	"
Strasburg	"	"	.920	"	"

Table from "De la Grandeur et de la figure de la terre," (24, p. 251) showing the relation of the various units to the French foot, considering the French foot as made up of 1440 parts.

Le pied de		Paris	of	1440	parts
"	"	Bologne	"	1682	"
"	"	Danemark	"	1404	"
"	"	Rhein ou Leyde	"	1390	"
"	"	Londres	"	1350	"
"	"	Suede	"	1316	"
"	"	Romain du Capitole	"	1306	"
"	"	Dantzik	"	1272	"
"	"	L'Amsterdam	"	1258	"
Le palme		Naples	"	1169	"
"		Genes	"	1113	"
"		Palerme	"	1073	"
"		Romain	"	990	"
La brasse de		Bologne	"	2640	"
"	"	Florence a terre	"	2430	"
"	"	Parme et Plaisance	"	2423	"
"	"	Reggio	"	2348½	"
"	"	Milan	"	2166	"
"	"	Bresse	"	2075	"
"	"	Mantoue	"	2062	"

As many of the descriptions which follow contain the French unit of dimension, we may note that a toise consisted of 6 French feet, each foot being made up of 12 inches and each inch of 12 lines. Approximately a toise equaled 6.4 English feet.

RICCOLI AND GRIMALDI.

About 1650, Riccoli and Grimaldi conducted a trigonometrical survey in Lombardy. Near Bologne, a base, in length 1094 Bologne paces and 2¼ ft. was measured; from a chain of triangles, the distance between Bologne and Modena was obtained. Instead of reducing to meridian distances, as had been previously done, they attempted to find the arc of the great circle passing through the two places, from the zenith observation of stars. The results for the value of 1 degree ranged from 56.130 to 61,797 toise.

Also, and it seemed to be the favorite method, they attempted to find the value of the degree by taking vertical angles to the different stations. These in connection with the distance between the stations and their respective heights above sea level, enabled them to obtain approximate values based on the curvature of the earth. From this method they obtained values of about 62,300 toises, for 1 degree.

These results have been rejected, because of the evident errors entering the work, for the methods employed are the most difficult of all. They call for extreme accuracy and, especially in the first case, knowledge of the true position of the stars, at that time not well known. Also in the triangulation work, all three angles of each triangle were not always measured, and some very small angles were used.

PICARD.

In 1669, Picard commenced his famous work, famous in that it was more accurate than any previously measured, for he established a much longer base line, and what was more important, introduced on his quadrant the telescope with spider lines. The comparatively straight highway, between Villejuive and Juvisy, furnished an excellent opportunity for a base.

The measuring rods, two in number, were made by joining end to end, two wooden rods, each two toise long, thus giving a rod four toise in length. In measuring the base, they were applied, end to end, along a stretched cord; at the end of every two measuring rods, a pin was placed in the ground, and having ten of these, they easily kept account of their measurements. The measurement of of the base forward, from the mill in Villejuive, to the Pavilion in Juvisy, gave 5662 toises and 5 feet; the backward measure gave 5663 toises and 1 foot. The mean of the two, 5663 toises, was adopted as the length of the base. By means of a system of thirteen triangles, the work was extended from Malvoisin on the south, to Sourdon on the north, and later to Amiens for the purpose of checking Fernel's work.

Triangulation. The angles were measured by a quadrant of a circle, of 38 inches radius, graduated to single minutes and provided with two telescopes, one fixed, the other movable, one end being hinged at the center of the quadrant, the other moving on the arc. As a verification of his work, a base of 3902 toises was measured, near the northern end of the system, and connected with it. The connecting line, as computed through the chain of triangles coming from the first base, was 6036 toises and 2 feet; as computed from the second base it was 6037 toises, a difference of 4 feet. In observing the angles, not only were the centers of the spires (for nearly all stations were towers of some sort) observed upon, but different parts, so that in case the center was inaccessible, a known point could be occupied. Many stations were reoccupied, in order to get the sum of the three angles to add up as near 180 degrees, as possible. Not all the angles of every triangle were taken, and where not observed, they were so taken as to complete the triangle; where all three were observed, and did not sum exactly to 180 degrees, the correction seems to have been applied to a single angle, that for some reason was considered as having the error, rather than being distributed equally among them. It is not exactly clear, how he reduced his observed angles to the values given since some must have been inclined angles, as it is highly improbable that all angles were in the same horizontal plane.

Astronomical work. The meridian zenith distance observations for latitude were made upon a star, in the constellation Cassiopeia, by means of an arch of $\frac{1}{20}$ of a circle, 10 feet in radius and graduated to $\frac{1}{3}$ minutes. Observations were taken at Malvoisin, Sourdon and Amiens. As a result, the difference in latitude between Malvoisin and Sourdon was found to be 1° 11′ 57″, and the corresponding meridian distance, from triangulation, when reduced to the latitude stations (for these were not the same as the triangulation stations) was 68,430 toises, giving 1 degree equal to 57,064½ toises. Between Malvoisin and Amiens the difference in latitude was found to be 1° 22′ 55″, and the corresponding meridian distance 78,907 toises, giving 1 degree equal to 57,057 toises. For a final value 57,060 toises was selected as the length of 1 degree.

Picard's work has also this honor, that Newton, by means of this distance, which furnished a close value as to the dimensions of the earth, was enabled to check the theoretical investigation of his law of attraction, which previous to this had been worked out, but laid aside because of the nonconformity of the theory, with the previously supposed size of the earth. We may note, that Picard neglected spherical excess in his triangles, which, for the size he used, would however have been very small; also, corrections to the position of the stars due to aberration, precession and nutation, errors at that time practically unknown. Nevertheless his results were quite close to the now recognized values, due partly to a balancing of errors, for slight errors have been found, in angles and calculations. Also the length of the base was in error by about $\frac{1}{1000}$, due probably to both errors in measurement and uncertain length of his standard.

7 Maupertius and his associates, in 1~~6~~39, after their return from Lapland, investigated Picard's work, reobserving angles, with the instruments used by them in Lapland, and obtained as a result, the meridian distance from Paris to Amiens 59,530 toises, with an amplitude of $1^{0}\ 02'\ 28''$, giving 1 degree equal to 57,183 toises or 123 toises greater than that obtained by Picard. The toise that Picard used, as a standard, was that of the grand Châtelet of Paris, 6 feet in length each foot divided in 12 inches, and each inch into 12 lines. Having a definite length, as a unit of comparison, was an improvement on all preceding measurements, for as we have seen, all were so indefinite as to be practically worthless. Picard's measurement, as a whole, was a step far in advance, and may be said to be the starting point for accurate determinations.

CASSINI.

About 1681, on the representation by the Royal Academy, of the results that would probably be derived from an extended survey, the King appointed several gentlemen to superintend the extension of Picard's work. M. Jean Dominique Cassini seems to have superintended the work from Paris southward, and M. De la Hire the work from

Picard's triangulation northward. The work was barely commenced, before it was interrupted, in 1683, by the death of the protector of the Academy, and was not again taken up until 1700.

During the years 1700-1718, the work was completed under the direction of Jacques Cassini, who had succeeded to the position of royal astronomer upon the death of his father. The triangulation was extended south to Spain and near Perpignan along the shore of the gulf of Lyons a base line of verification was measured.

Base apparatus. The measuring rods, after the fashion of Picard, were made by joining together two wooden sticks, each 2 toises long, giving a rod 4 toises in length, the ends of which were capped with copper, and the whole adjusted as accurately as possible, to a standard length, by means of an iron rod 4 toises in length, which had been brought from Paris. The base line was first aligned, and with a string 10 toises in length, large stakes were driven to mark every 100 toises. Where the ground was rough, the measuring rods were supported horizontally, by means of vertical staffs provided with horizontal arms. Elsewhere they seemed to have measured along the ground. The base line crossed six brooks or marshes where it was difficult to measure, and here they resorted to triangulation. Usually they turned off a right angle, and measured on this such a distance that when, from this new point, an angle of 45 degrees was turned, it would intersect the base line, at some point across the river. The measured distance would then be the base line distance, across the river. When they did not employ this method, they measured a distance along the river, observed all the angles of the triangle and computed the distance across.

The base line is subject to serious criticism, on account of being broken so many times. Signals made of trees were erected at terminal stations, which, on account of their location near the sea, were often shut in by fogs. To facilitate the taking of angles at one extremity of the base, a signal was erected on line about 1,000 toises from the other extremity, where from its location it was less liable to be enveloped in fogs, and from this the angles were measured. As a whole, we must consider this as a rather poor base line to check upon. The length of the base as computed from the triangulation extended south from Paris, was three toise less than the measured length. Later, by allowing for errors of the instrument and reductions of angles on account of their being in different planes, they brought the final computed result only a little smaller than the measured result. This reduction of angles to the horizontal should have been made previous to any attempt of comparison.

Instruments. For observing the angles of the triangles, a quadrant of 39 inches was used. This was graduated to minutes, and had two telescopes, one fixed, the other movable. This was after the manner of Picard's instrument, differing only in the length of the radius. Also an octant, if it can be called such, was sometimes employed. This was peculiar in that it had three telescopes, two fixed and one movable. Of the two fixed, one formed a radius of the arc, and the other was fixed perpendicular to the first. The extent of the limb was 40 degrees, so that in use, for angles less than this amount, the fixed telescope along the radius and the movable one was used; for angles greater than 40 degrees, the other fixed telescope and the movable one were employed. For observations on the stars, for zenith distances, an arc of a circle ten feet in radius, graduated to degrees and minutes and of 26 degrees extent, was used. The observations at the southern extremity were made at Collimere in March, 1701; the northern observations at Paris in March, 1702, the same star being used at both places. The meridian amplitude, allowing for refraction and change in declination, was 6° 18′ 57″, and the computed distance between the astronomical stations, when reduced to sea level, gave as the value of one degree, 57,097 toises. This was greater than the value that Picard obtained by 37 toises, and led to the conclusion that the earth was elongated at the poles.

The triangulation was then extended northward from Paris, using nine of the triangles that Picard had established, and adding twenty more, thus extending the work to Dunkirk, on the Straits of Dover. At the extremity near Dunkirk, a northern verification base was measured along the sea-shore. Here the work was aligned by a telescope and wooden measuring rods three toises in length were used. Where the ground was irregular over the sand-dunes, the rods were held horizontal and the extremities plumbed down, stakes being driven and the points marked from which they proceeded; a few places over spots of water were measured by a stretched cord. Some 4000 toises of the base was remeasured, giving a difference of three feet from the first measurement. The final value of the base-line was given as 5564 toises. The computed value from the triangulation agreed within one toise. The star observations were made at Dunkirk in 1718, using the same type of sector that Picard used, namely, ten feet in radius, graduated to thirds of minutes, and twelve degrees in extent.

The amplitude of the meridian arc, as given by the observations, was 2° 12′ 9.3″, and the corresponding distance 125,454 toises, giving one degree, equal to 56,960 toises, which was smaller by 100 toises than Picard's value. This served to strengthen the belief of some that the earth was elongated at the poles.

Newton had announced, after the law of attraction had been established, that from analytical principles the earth must be a spheroid flattened at the poles, which view was agreed with by Huyghens and others. The seeming conflict between theory and results led to the establishment of two schools, those who followed Newton's view and those who accepted the Cassinis' opinion. Between the members more or less discussion arose, which was not lessened, when in 1734 an arc of a parallel from Strasburg to St. Malo was measured, which seemed to strengthen the view of the Cassinis.

THE LAPLAND ARC.

To settle this question of whether the earth was an oblate or a prolate spheroid, the Royal Academy of France, which thus far had been the leading spirit in arc measurements, suggested that measurements be made under as varying latitudes as possible. As a result of this suggestion the Academy was requested by the King to undertake such measurements. Accordingly two expeditions were sent forth, one to Peru and one to Lapland. We will first describe the Lapland expedition, for although it did not start for nearly a year after the Peru party, it was the first to return and make known its results.

The party consisted of four members of the Academy, Messrs. Maupertuis, Clairaut, Camus, and Le Monnier, who were also accompanied by M. L'Abbé Outhier and M. Celsius, professor of Astronomy at Upsala. The party, which was stronger in a scientific sense than the Peru party, was under the general direction of Maupertuis, and all members worked together in the utmost harmony. According to the journal of Outhier, who kept a daily record of the journey, they left Paris on Friday, April 20th, 1736. Their objective point was the northern extremity of the Gulf of Bothnia, for from consulting the existing charts they hoped to use numerous islands in their plan of triangulation. They first went to Stockholm and while there were presented to the King of Sweden; from thence they proceeded to Torneo at the mouth of the Torneo River, at the northern extremity of the Gulf of Bothnia, arriving here in July, 1736.

A reconnaissance showed them that their proposed plan was not feasible; the islands could not be utilized as they possessed no height. From inquiry and investigation they found that the river Torneo ran almost due south and that the mountains or either side might afford a scheme of triangulation. Either this plan of triangulation must be adopted, or they must go south to some favorable point in Sweden. It was decided to attempt it, and they made a beginning, not knowing whether it would be possible to establish a good system or to measure a base line that would connect satisfactorily. The difficulties they encountered were enough to stop the work of any but those filled with an enthusiastic spirit of scientific investigation, for of conveniences they had none, camping on the mountains affording poor lodging and poor fare. The mountains were steep, wooded and difficult to ascend, while the rivers and streams traversed were filled with rocks and rapids. Prevailing fogs delayed them sometimes for a week, and the flies or gnats, which appeared in swarms formed such an intolerable pest that the only way to be free from them was to sit in a thick smudge of smoke, or cover one's face and hands with pitch. With the assistance of soldiers stationed near, and with the services of the natives they were enabled to conquer all these difficulties.

They divided up the work, some making reconnaissance, others erecting signals and still others ascertaining the direction of the lines. With the assistance of the soldiers the mountain tops were cleared of timber and the signals erected. These were made by joining together in the form of a large hollow cone a number of trees peeled of their bark and left white, thus making a signal that was plainly visible. Marks of reference were made on the rocks or on stakes driven, and the center of the signal located from these, so that in case of accident the exact position might be recovered. This was fortunate, for upon one of the mountains a fire broke out after they had left, probably starting from the embers of their camp fire, which burned the signal and necessitated its reconstruction. The signals, being constructed as they were, afforded ample room for setting up the instrument directly over the station without taking down the signal.

Instruments. The angles were measured by a quadrant two feet in radius, provided with a micrometer; frequent verifications of the quadrant were made by taking the angles around the horizon and the result always came very close to four right angles. In observing the angles the quadrant was always placed over the center of the station and each observer wrote down his observation separately; afterwards they were compared and the mean taken, for they differed very little. Also on every mountain they were very careful to observe the elevation and depression of all the signals between which angles were taken, and the reduction of the angles to the level of the horizon was worked out on the mountains before leaving the stations. All three angles of each triangle were observed, as well as other angles, which by their sum or difference would give checks on the work.

The sum of all the angles of the triangles amounted to 1.97″ more than the required number of right angles and is remarked upon by Maupertuis, as agreeing with theory. This seems to be the first recognition of spherical excess. The reconnaissance, erection of signals, and taking of angles, had been accomplished in 63 days.

Astronomical work. They then commenced upon the astronomical work, starting first at Mt. Kittis, which was their most northern point. Here they arrived September 9th., and having bought from a native a building, they proceeded to take it down, and carrying it to the top of the mountain erected two observatories there. The smaller observatory was built on the site of the signal and contained a clock, the quadrant previously described and a small transit instrument with a telescope of 15 inches in length. This transit instrument, made by Graham of London, was placed directly over the spot which was the center of the signal and was used for obtaining the direction of the triangles with the meridian, or in other words, the azimuths of the lines of triangle at Kittis. The larger observatory, situated so near that the clock beats in the smaller observatory could be counted, contained the large zenith sector, also made by Graham.

This sector (15, p. 6) "consisted of a brass telescope 9 feet in length, forming the radius of an arc of 5° 30′, divided into spaces of 7′ 30″. The telescope, the center from which the plumb-line was hung, and the divided limb were all in one piece, the whole being suspended by two cylindrical pivots which allowed it to swing like a pendulum in the plane of the meridian. One of these pivots ended in a very small cylinder at the exact center of the divided limb, and at its plane formed the suspension axis of the plumb-line. The plumb-line was suspended from a groove in such a manner that it was obliged to fall into the vertex of the groove, and for verticality of the instrument was brought by the help of a microscope to pass over a fixed point near the base of the instrument. The divided limb had a sliding contact with a fixed arc below, and this arc carried a micrometer against the pivot of which the limb of the sector was kept pressed by the tension of a thread. This micrometer screw, by communicating to the telescope and limb a slow movement in the plane of the meridian, served to subdivide the space graduations of 07′ 30″." The pedestal carrying the telescope was 12 feet in height.

Observations were commenced Sept. 30th. The time-piece was regulated daily from solar observations. With the small transit instrument the time of the sun's passing the verticals of Niemi and Pullingi, which were the other two stations forming the triangle with Kittis, was taken, whence, knowing the latitude of their station, the position of the sun and its hour angle, they were able to compute the azimuth of these lines. Eight such observations, the results of which differed by less than a minute, gave as a mean of the azimuth of Kittis to Pullinger, 28° 51′ 52″. For work with the sector, the star δ Draconis was selected as it passed about a degree from the zenith, and the greatest variation of results by the different observers was 3″. The observations were completed by the 23rd, and all members of the party had left by the 25th, arriving at Torneo the 28th. Nov. 1st, observations were commenced here at Torneo giving results which differed by only one second. The observations at both places were made in the daytime and no artificial light was necessary to illuminate the cross hairs. Taking the mean of the observations, after reducing the parts of the micrometer to seconds, correcting for change in declination due to the elapsed time, precession and other causes, gave the amplitude as 57′ 26.93″.

The base line, which was near the middle of the system of triangulation, had purposely been selected so as to pass over the river, in order that it might be measured upon the ice. The early part of December was spent in making the measuring rods. They had brought from France an iron toise, which later became known as the "Toise of the North." This, with the one taken to Peru, had been adjusted at a certain temperature before leaving Paris. By means of this toise eight measuring rods, each five toises in length, were constructed: First five wooden rods, each one toise in length, were made by taking pieces of fir and driving into their extremities large nails and filing down these nails until the rods were the same length as the standard toise; which was accomplished by having a frame into which the standard toise fitted, and then fitting them to this frame. So closely were they made to fit that Maupertuis says it was impossible to insert the thinnest piece of paper. With these five wooden rods, a standard of 5 toises in length was made by inserting in the walls of the building two nails at nearly the proper distance and filing them down so that the five wooden rods, when in contact with one another and supported horizontally, just fitted the space between. From this standard they constructed, after the manner of the single toise, light wooden measuring rods with iron nails at their ends. During the process of construction, the standard toise was kept in a room heated to the temperature of 14° Réaumur, which was the temperature at which it was standardized. The measuring rods after construction were subjected to many experiments as regards changes in temperature and were found to differ far less than the standard toise, so that they considered as inappreciable the changes in them due to temperature.

The measurement of the base was commenced at the southern end on Dec. 21, 1736. Snow having fallen to the depth of two feet since the river had frozen, they attempted to clear it from along the base line by making drags of logs and having them drawn by reindeer. This method of removing the snow was unsuccessful and probably the measurement was made by laying the rods on the snow.

The measurement was made by separating into two parties, each taking four rods; and these being numbered, they were careful to place them aways in the same order of contact. The first day they measured 700 toises, for only four or five hours were available for work, since the sun just barely rose above the horizon at noon. But the long twilights, the whiteness of the snow, and the numerous fires, afforded them the necessary light for the above length of time. For six days they continued their measurement amid the extreme cold and snow, and then a recess of one day was taken in which the remainder of the line was prepared while Maupertuis and Outhier ascended the mountain of Arosaxa to take the measurement of the height of the signal, which they had forgotten to do when there. The next day the measurement was completed, the two different lengths checking within 4 inches in the total length of 7406 toises and 5 feet. Most of this difference occurred on the last day, for previous to that time they had not differed by more than 1 inch; but the last day was exceedingly cold, the thermometer standing 37 degrees below zero on their return. The line was also checked by measuring it with a cord of 50 toises in length, to make sure no error had entered.

They remained at Torneo during the winter and made their calculations; for knowing the length of the base line they were able to compute the other distances. Since they made no provision that the length of the side should be the the same in whatever way computed, the meridian lengths varied according to the method of computation. The greatest difference that could result from any arrangement was 54 toises. They finally fixed on two methods which they deemed best and the result differed by only 4 toises, the meridian distance between Torneo and Kittis being given as 55,023.5 toises. The value of one degree from these results differed about 950 toises from what they had expected to find. This led them to decide on remeasuring the astronomical amplitude and to investigate carefully their sector.

The arc of the sector, 5° 30′, from information furnished by the maker, was small by 3¾″; this they investigated themselves by mounting horizontally and laying off from the center of the sector a right angled triangle very carefully measured. From the measurements, the angle at the center was 5° 29′ 50″ and the instrument read 5° 29′ 48.95″, but this is claimed by Clarke (15, p. 9) to be a misprint as it should be 5° 29′ 52.7″, thus agreeing substantially with the maker. The 7½′ spaces were all measured with the micrometer, and especially the parts where the reading of the stars came.

The astronomical remeasurements were made in March at Torneo, and in April at Kittis, on the star α Draconis, which was nearer the zenith than the one previously used. Three observations at each place gave as a resulting amplitude 57° 30.42′, a difference of 3½″ from the first. Allowing for the error in the sector and the inequality of the graduation due to the observations on the different parts of the limb, this difference was reduced to 2½″, giving as the mean amplitude 57′ 28.67″.

During May re-observations for azimuth were made at Torneo by taking angles between signals and the sun at its rising and setting, and noting the time. From calculations the azimuth was obtained and this differed 34″ from previous results at Kittis; not enough to make any material difference in the previously calculated length of the arc. Taking then the length as 55,023.5 toises and the amplitude as 57′ 28.67″, the resulting value of one degree was 57,437.9 toises or 377 toises greater than Picard's value. This proved that the earth was an oblate spheroid and led to the remark by Voltaire that Maupertuis "flattened the poles and the Cassinis." (52 Vol. 1–p. 100) In fact the result was larger than theory led them to expect.

Maupertuis seems to have understood and allowed for, as far as possible, all theoretic corrections, except the reduction to sea level, and in this case the correction would be very small. Because of the difference in astronomic determination, and the later determination of new instrumental corrections and the difference from theory, the results were questioned in later years, notwithstanding the seeming accuracy of the work. A remeasurement was made which accorded more with theory, which will be discussed later.

THE PERU ARC.

The party assigned to the equatorial measurement consisted of three members of the Academy, Messrs. Godin, Bougeur, and De la Condamine, and their assistants. They left France May 16, 1735, sailing to Martinique in the West Indies; thence to Carthagena in Columbia where they were joined by two Spanish officers, Don George

Juan, and Don Antoine de Ulloa. On account of their proposed work being in Spanish territory, these two officers had been delegated by the King of Spain to accompany them. From Carthagena they sailed to Portobello, then crossed the Isthmus to Panama, arriving December 29th, 1735. On their journey along the coast and across the Isthmus, they made observations for latitude and refraction, and studied in general the geography and interesting features of the country. Leaving Panama February 22nd, 1736, they sailed to Manta on the western coast of Ecuador. Here Bougeur and De la Condamine remained investigating refraction, making astronomical observations and a general geographical study. Having determined the position of the equator, they had chiseled upon the rocks an inscription to that effect. Leaving here they separated, perhaps in order to make a better geographical study, and by different routes journeyed to Quito, whence the other party had preceded them. By June 10th, 1736, all had again reassembled. Some delay was experienced in obtaining their equipment, which had to be brought by Indians. The time until autumn was spent in making latitude observations and a general reconnaissance of the country, in order to locate a proper place for a base and its extended system.

The valley in which Quito is situated runs nearly north and south and is hemmed in by very rugged mountains on either side, affording a good system for triangulation. The plain of Yarouqui north of Quito was selected as being the best suited to base measurements.

Base measurements. The measurements were made independently by two parties working in opposite directions; one party consisted of Godin, Juan and assistants, the other of Bougeur, De la Condamine and assistants. Each party was provided with three wooden measuring rods each 20 feet long and tipped with copper; these rods were painted different colors in order to be easily distinguished, and were always used in the same order. The measurements were made horizontal and a plumb line was used where it was necessary to change from one horizontal to that of another.

The measurements made between October 3rd and November 3rd, 1736, gave as a result

6272 toises 4 feet 5 inches by Bougeur's party
6272 " 4 " 2½ " " Godin's "

This, when reduced to the level of the lower end of the base line, was 6,272.6559 toises. During the measurement, the rods were daily compared with a field standard that had been made and compared with the one brought from France. The remainder of the year 1736 was spent in verifying their quadrants and in making observations on the sun for the azimuths of their triangulation lines.

The instruments used for measuring the angles of the triangles were quadrants of varying radii, that of the Spanish officers 24″, that of M. Godin 21″, and that of M. De la Condamine 4 feet. These seem to have been faulty in construction, and much time was spent in making experiments for the detection of errors of graduation, especially by M. De la Condamine, who investigated his quadrant very thoroughly, preparing a table of corrections for every degree of graduation. The angles in general were measured by three different parties; 1st, M. Godin and Don George Juan, 2nd, M. Bougeur and Don Antoine de Ulloa, 3rd, M. De la Condamine.

During the year 1737 the triangulation was commenced but not very much was accomplished on account of the difficulty and time it took to climb the mountains and erect the signals. Also the first location for the signals was, after a few angles had been measured, thought to be not the best, and relocations were made necessitating re-observations. About one half of the year, May to November, was unfavorable for work owing to the rainy season.

Signals. The signals first used were pyramidal in shape, constructed out of wood and covered with mats of straw or native cloth. These gave poor service, as after the weeks of continuous rains and high winds they were often found blown down, and the work much delayed. For a substitute they used their tents, of which each party had one; and as the parties were not together, a tent often served for the shelter of one party and at the same time a signal to the others. When not occupied by parties the tents were left in the care of native Indians. In this way much of the work was done, the angles of each triangle being measured by all three parties. Triangulation was the main work of the year 1738, although a number of observations were made on the sun for azimuth.

The observations were completed in the first half of the following year, and in August, 1739, the verification base at Tarqui was measured. This base was measured by Bougeur and assistants working from south to north, and by De la Condamine and assistants working from north to south, after the manner employed on the northern base. The measurement agreed within one foot and one inch, which, when corrected by comparison of the measuring rods with the field standard, brought the agreement between the two to about one inch and gave as a mean result, 5,259.20 toises. This became 5,258.949 toises when reduced to the level of the northern base. The observed angles were reduced to the horizontal and to the center of the station, for nearly all observations were made eccentrically. Correction was also made for inclination of signals. The length computed from the chain of 33 triangles, starting at the northern end, differed between 3 and 4 feet according to Bougeur and about 6 feet according to De la Condamine, being 5,260.03 toises.

The meridian distance between the terminal stations was computed, and because the computed value of the verification base was large by about $\frac{1}{5000}$ of itself, De la Condamine decreased this computed meridian distance value by $\frac{1}{10000}$ of itself, giving as a result 176,958.44 toises. This was further reduced to the elevation of the northern station, giving as the meridian distance, at an elevation of the northern base, 176,950.47 toises.

Astronomical work. The astronomical observations were commenced at Tarqui in December, some time having been spent in reconstructing their sector, which was 12 feet in radius with an arc of 50 degrees. For this one of 5 degrees was substituted. The instrument was carefully tested for errors of graduation and value of the micrometer. Observations at Tarqui were completed in June, 1740. Observations at Cotchesqui were made between February 19 and April 25, 1740. From the amplitude given, the value of the length of one degree came so different from what was expected, that they believed the amplitude to be in error. Accordingly observations were made at Tarqui by Bougeur between March 5 and December 4, 1741, but still the results were doubtful and rejected. For the third attempt, they decided on methods which they hoped would eliminate as far as possible all errors; namely, to make simultaneous observations at each place which would eliminate any unknown changes in the place of stars. During the period November 1742 to April 1743, M. De la Condamine observed at Tarqui and M. Bougeur from November 1742 to June 1743, at Cotchesqui on E Orionis. The sectors were reversed during the observing, thus giving double zenith distances and eliminating errors as far as possible. Taking the observations that were made on the same nights and applying all correction due to instrument, refraction, etc., gave as a resulting amplitude 3° 07′ 1.0″, which was the same as by taking the average of the two series.

Using then 3° 07′ 01.0″ as amplitude and 176,950.47 as the meridian distance gave one degree equal to 56,770.2 toises at an elevation of the northern base. Correcting for the height of this, 1226 toises above sea level, gave one degree equal to 56,748¾ toises at sea level.

There seems to have been a lack of harmony among the members of this party, for as has been noted in the preceding description, much of the work was done individually, hence three different reports were given with varying results.

For the value of			1	degree	Bougeur	gave	56746
"	"	"	" 1	"	De la Condamine	"	56749
"	"	"	" 1	"	Ulloa and Juan	"	56768

Their results still further checked those of the Polar party and forever settled the question of the earth's being an oblate spheroid.

During the year 1740, monuments were erected to mark the terminals of the base at Cotchesqui, chiefly through the labor of De la Condamine. These have since been destroyed.

THE MERIDIAN OF PARIS VERIFIED.

During the years 1734 to 1742, nearly all France was covered by systems of triangulations, chiefly due to the labors of Jacques Cassini, Cassini de Thury, Maraldi and Lacaille. During these eight years, 18 bases were measured and some 400 principal triangles used. In the earlier years the work was in connection with the establishment of a perpendicular to the meridian of Paris. The bases measured were as a rule short, only one reaching five miles in length. The work of the middle period was principally in western and southern France and the bases were about six miles in length. The last part of the period was devoted to revising the meridian through Paris and the results were published in 1744, under the title *La Méridienne de l'Observatoire... ....vérifée.* In connection with this work, six new bases were measured. Taking them in order from north to south they were as follows.

Dunkirk base, 6224⅓ toises in length, was measured twice with wooden rods. This base was on the sea shore and crossed the entrance to the harbor. The measuring rods used were after the style of previous rods, and since all but one of the bases was measured with wooden rods, a general description can be given. They were made of wood, either 18 or 24 feet in length, 3 or 4 inches broad and 2 inches thick, and were dipped in oil. The extremities capped with iron. They were placed in terminal contact and were compared three times daily with 4 iron rods, each 3 feet in length, that had been compared at the standard temperature, 14 Réaumur, with a line of 10 toises laid off in the hall of the Paris observatory. The mean of the differences of comparisons with the field standards was applied as a correction to the wooden rods, and probably in making this correction allowance was made for the change in length of the iron field standards, due to temperature. The part of the Dunkirk base across the harbor, was obtained as in previous like cases, from a right angled triangle whose acute angles were 45 degrees.

Amiens base, 5,242 toises 4 feet, was remeasured, checking within 4 inches.

Paris base, 5,749 toises long, was measured five times by means of four iron rods, each 15 feet long. The measurements checked between one and four inches.

Bourges base, 7492 toises in length, was measured on an incline and later reduced to horizontal.

Rodez base, 4422 toises long, was also measured on an incline and reduced.

Perpignan base, 7929 toises long. The stations as a rule were towers and churches and, in some few cases, erected signals. The angles were measured with a quadrant of two foot radius and reduced to horizontal and center of station. In calculation, the angles were adjusted so as to make the calculated bases and azimuths agree with the measured bases and azimuths. In making this adjustment, no correction of over 5″ was necessary.

Astronomical observations were made at five places, Dunkirk, Paris, Bourges, Rodez and Perpignan, using a sector of 6 feet in radius and an arc of about 50 degrees, fitted with a micrometer. This was tested by taking a round of angles to close the horizon and the equality of division noted by taking the same angle on different parts of the limb. Observations were made on αLyrae, αCygni, αPersei, γ Draconis and ηUrsae Majoris, each star being observed eight or ten times at each station, the sector being reversed during the progress of the observations. As a result of the triangulation and astronomic work, there was obtained the following,

Arc, Dunkirk to Paris,	meridian distance 125,431 toises, amplitude 2° 11′ 50.28″ giving 1 degree equal to 57,084 toises.
Arc, Paris to Bourges,	meridian distance 99,990 toises, amplitude 1° 45′ 7.33″ giving 1 degree equal to 57,071 toises.

Arc, Bourges to Rodez,	meridian distance 155,767 toises, amplitude 2° 43′ 51.5″ giving 1 degree equal to 57,040 toises.
Arc, Rodez to Perpignan,	meridian distance 94,308 toises, amplitude 1° 39′ 11.2″ giving 1 degree equal to 57,048.5 toises.

The results probably were fairly accurate for the time, as all corrections to the observations, such as precession, aberration and nutation, were applied. These results decrease in going from south to north, except in the last value between Rodez and Perpignan. The value between Bourges and Rodez appears smaller than would be expected from a comparison with the first two, so that we may believe the last two to be slightly in error.

THE ITALIAN ARC.

During the years 1750 and 1751, the Jesuit fathers, Marie and Boscovich, under papal authority, conducted a survey in Italy having two objects in view; 1st., to determine the figure of the earth, and 2nd., to make a map of the Papal states. The preparations for the work of the first part were carefully made. In order that their results might be compared with previous work, Boscovich requested that a French toise might be sent them as a standard. Accordingly an iron toise was made under the direction of M. de Marian and sent to them. This served to compare their units of measure with the prevailing standard. These relations as given by Boscovich (42, p. 358) are

Roman measure	to the foot of Paris	to the toise
The palm	as 2971 to 4320	as 2971 to 25920
" foot	" 2971 to 3240	" 2971 to 19440
" pace	" 2971 to 648	" 2971 to 3888

The measuring rods employed, three in number, were 27 palms (about 18 feet) long, 3 inches wide and 2 inches thick. They were called perches. Each perch had upon its top surface four strips of copper, each bearing a slight mark, and placed so as to form three equal intervals of 9 palms each. These perches were marked 1, 2 and 3, and were always used in this order during the measurement. The base was measured horizontally, the perches being supported upon heavy wooden tripods. The tops of these being attached to a vertical sliding rod, could be placed at the desired elevation and held firmly by means of an abutting screw. The rods were not brought into contact for fear of slightly disturbing their position. They were placed in line with their ends nearly touching and the distance between the terminal marks on the copper strips of two adjacent perches measured by means of beam compasses. Where the ground was rough and the rods were laid horizontal, but at different heights, the end of one was brought over the end of the other by having a plumb line just touch the end of each rod.

In the process of the measurement, the perches were compared several times during the day with the iron standard. This was done by having marks graduated upon the standard, 9 palms apart, which distance was taken off by means of beam compasses and compared with the same distance between the copper strips of the perches. In this way changes due to the temperature were detected and allowed for. Allowance was also made on the Rimini base for curvature of the rods between the supports. This was obtained by stretching a cord between the end marks of the perches and noting the deflection at the other marked points.

The base near Rome was measured along the ancient Appian Way, at that time neglected and abandoned. This way, straight for about ten miles, and lined on both sides with ancient tombs, afforded ease both in measurement and in alignment. The starting point was opposite the inscription on the tomb of Mettala, the terminal opposite that of Frattochie. This latter point was marked by a large stone sunk in the ground. The terminal of each day's work was marked by driving a stake into the ground and marking a point upon it by plumbing down. This Rome base, measured once during April and May 1751, gave a length of 5356½ palms or 6139½ French toises. The base at Rimini was measured in December 1751. The base was aligned the first day. This proved fortunate, as the weather became foggy, although not enough to obscure the view of the alignment stakes. As a result of the weather, the base was measured at nearly a uniform temperature. The humidity of the air, however, produced noticeable changes in the lengths of the perches.

This Rimini base was a broken base, the angle between one of the measured sides and the computed side being 9° 07′ 45″. It also crossed a river, the distance of which was obtained by triangulation. The sides of the broken base were measured twice, and differed by only two inches. The resulting computed rectilinear base was 6,037.62 toises.

The triangulation system consisted of 9 main triangles, with a maximum side of about 50 miles, and 2 auxiliary triangles. The signals were constructed of trees placed either in pyramidal or conical form, being about 15 feet in diameter at the base, and were easily seen against a sky background. In case they came against some higher mountain, they were covered with white cloth to render them more conspicuous. The angles were observed eccentrically, a quadrant three feet in radius, made by M. Ruffo of Verona, being used. This had the customary two telescopes, one fixed, the other movable, and was graduated by 10 minute division and read directly to minutes by means of a transversal crossing 11 concentric circles marked on the limb. The seconds were obtained by the use of a micrometer. The whole was mounted upon a heavy vertical standard, with connections that allowed the sector to be placed in any desired plane. Verticality for measuring the angles of elevation or depression to the different signals was obtained by the use of a plumb line hanging parallel to the plane of the sector.

Astronomical work was done at Rome and Rimini. At Rome, the azimuth was determined by use of the quadrant from taking the angle between the triangulation station Sorianio and the setting sun. Three observations were made, with a maximum range of 25″, and the mean of the three taken. At Rimini, three observations were made on the rising sun, with a maximum range of 45″, and their mean taken. The amplitude of the arc was determined from observations on α Cygni and μ Ursae Majoris.

At Rome, observations were made on six different nights, four with the face of the sector toward the east, two with it toward the west. The results corrected for refraction, precession, nutation, etc., were

zenith distance of α Cygni	2° 30′ 20.7″
" " " μ Ursae Majoris	50′ 00.8″

At Rimini observations were made on seven different nights, the sector being reversed as before. The results were

zenith distance of α Cygni	20′ 34.6″
" " " μ Ursae Majoris	1° 19′ 46.6″

The resulting amplitude was then

from α Cygni	2° 09′ 46.1″
" μ Ursae Majoris	2° 09′ 47.4″

The former amplitude was selected as probably being nearer the correct value. The instrumental errors between the two times of observation at Rome and Rimini were found to cause a change of about two minutes on account of a small disarrangement of the objective of the telescope. (42, p. 152) Since the sector was reversed, this error would be practically eliminated.

These astronomic observations were made with a sector nine feet in radius. It consisted essentially of an iron bar of T shape and suspended inverted. The vertical part had rigidly attached to it the telescope, while upon the horizontal part was placed a limb of copper having a movable slide graduated into 72 parts and fitted with a micrometer screw, the head of which was graduated into 180 parts. The angles were thus measured from a line of equally divided tangents. Provision was made for a small motion in azimuth in order to bring the sector into the plane of the meridian.

The meridian distance between the latitude stations was 161,253.6 paces, Taking the amplitude of the arc given by the star αCygni, namely 2° 09′ 46.1″, the value of one degree would be 74,557.6 paces or 56,972.9 toises; and from μ Ursae Majoris, amplitude 2° 09′ 47.4″, the value of one degree would be 74,545.2 paces or 56,963.4 toises. Since the computed value of the Rome base was about $\frac{1}{8000}$ less than the measured value, the value 56,972.9 was increased in the same proportion and the value of one degree taken to be 56,979 toises in the mean latitude 43° N. The work was very well done, although it has since been found that local attraction affected it to a certain extent.

This value of one degree should be diminished by $\frac{1}{25000}$ of itself, for in comparison with later standards it has been found that the toise of Marian was short by that fraction of the proper length.

LACAILLE.

In the year 1752, M. Lacaille, astronomer at the Cape of Good Hope, made, in connection with his astronomical work, a meridian arc measurement. A base line near the middle of his system on the plain of Swartland was measured, October 17 to 21, 1752. The ground for the base line was quite even but had several places covered with brushwood; the base was cleared and aligned in seven days.

An iron standard toise had been brought from France; from this another one was made of wood, tipped with iron and standardized. From these two, four measuring rods of fir, each 18 feet long, 3 inches wide and 2 inches thick, were constructed. These were capped with iron and well painted. During the measurement, the two one toise rods were kept in the field as standards and were well sheltered. On the first three days of the measurement the sky was clouded and a south wind was blowing; this increased and blew strongly on the last two days. But no sensible change was noted in the length of the iron toise during the measurement.

Concerning the measurement Lacaille says "I commence the work of the morning by assuring myself of the true length of the four rods aligned and placed end to end. I measure then 600 or 700 toises according as the ground is more or less even. I take upon myself the care of making exact contact between all the rods. For each part which was of 12 toises, I give myself a counter, as representing the head of the measuring rods. I place a small stake almost level with the ground at the end of 10 parts or 120 toises. I take up then the measurement a second time, and in always counting my counters I see if at each ten I fall again on my small stake. Having arrived at the end where I started at first, I mark the difference between my two measures; then I verify the length of my four measuring rods. In like manner was the work of the morning. I then have transported my measuring rods, bed and provisions (for we were in a desert) to 1200 or 1300 toises farther ahead, that is to the place where I expect to terminate the measure of the day. Then, after having dined and taken some rest, I determine the length of my four measuring rods, I proceed in measurement as far as the point where I had stopped at midday, I take up a remeasurement, and conclude by verifying again the length of my four measuring rods."

In this manner Lacaille found for the measurements 6467 toises 4 feet 3½ inches, and 6467 toises 4 feet 11½ inches, a difference of 8 inches. Taking the mean and correcting by 2 feet 9⅓ inches, to allow for inequalities of level of the ground, gave 6467 toises 1 foot and 10 inches. This was called 6467¼ toises in round numbers, allowing for small errors that may have entered. The scheme of triangulation contained only four triangles, a very small number for the resulting amplitude.

The angles taken on the large rocks which formed the stations or upon fires built at the stations were observed with a quadrant of three feet radius fitted with a micrometer. These angles, observed eccentrically, were reduced to the station and horizontal. The meridian distance from Cape Town, the southern station, to Klipfonteyn, the northern station, was 69,669.1 toises. For the astronomical work the sector used in 1738 in the verification of the meridian of Paris was used, being of 6 feet radius, and arc of about 50 degrees, fitted with a micrometer. Observations were made at Klipfonteyn, September 16th, 18th, and 19th, 1752, with the sector in one position and on the 22nd, 23rd, and 24th, in the reversed position, sixteen stars being used.

The azimuths of the lines were observed at Cape Town in August 1752, from seventeen observations on the rising sun. The star observations at the southern end having been made the previous year, the amplitude after reducing the stars to the same time and applying corrections was 1° 13′ 17⅓″. This gave 1 degree equal to 57,037 toises at a mean latitude of 33° 18½′. This result was unexpected, as it made a degree at this south latitude equal to a degree at a point ten degrees greater in north latitude, showing that the figure of the earth was not exactly symmetrical.

To make sure that there were no errors in the base, Lacaille remeasured it November 2nd, 1752, with a cord of 30 toises in length, and he concluded that there was no error. Later the amplitude was changed by him to 1° 13′ 17.5″ giving 1 degree equal to 57,034.4 toises. Because of the short arc it has sometimes been given less credit than it should be, for it was very well measured, as was found by Sir Thomas MacLear, who remeasured it in 1838.

LIESGANIG.

The Vienna astronomer, Liesganig, in connection with the establishing of the meridian of Vienna in 1762, determined a meridian arc. The first base, near Neustadt, was 6238 toises long; the verification base, in Marchfeld, was 6388 toises.

The unit of measurement was the Vienna fathom carefully compared with the French toise. The measuring rods were of wood, six fathoms long, made by splicing together several pieces. In measuring they were placed in end to end contact along the ground. The triangulation angles were measured by a quadrant 2½ feet in radius which was carefully tested. These were reduced to the horizontal. The sector was like the one just described, except that it was 10 feet in radius. Numerous star observations were made and the sector reversed during the work. The meridian distance of the extremities of the arc from Sobieschiz on the north, to Varasdin on the south, was 172,996 Vienna fathoms, and the amplitude $2° 56' 45.5''$, giving 1 degree equal to 58,655 Vienna fathoms or 57,077 toises.

The work was apparently exceedingly well done, but it has since been found to have contained a serious blunder. A certain star designated by him was not that star but another; this however causes no serious error for it was used at both stations, and the only error would arise from uncertainty of position and reduction. The blunder came in at the southern extremity and was probably due to mistaking a signal, for it has since been found that the position of stations are in error as much as 2,500 toises. Also errors in angles have been found. On account of this blunder no weight can be given it and it has been discarded.

A second arc measure was also obtained between Kistelech as a northern point and Czuroch as the southern. This meridian distance was 59,990 Vienna fathoms, and its amplitude $1° 01' 34.5''$. Hence 1 degree was equal to 58,453 Vienna fathoms or 56,881 toises. Many small triangles entered into this work and criticism on other grounds has arisen, so that little credit is given to it.

BECCARIA.

In 1762-1764, Beccaria, probably at the instigation of Boscovich, measured an arc in Lombardy. The whole work of Beccaria lay in a flat country, while that of Boscovich was in a mountainous region. Hence it was thought that the results of the two ought to give a true approximation to the real value of one degree. The base line was measured with wooden rods supported on a trestle, except two small parts that were determined by triangulation. The measuring rods were made from comparisons with an iron toise which had been compared by the French authorities with the toise of Peru. The length of the base was 6501 toises.

Instruments. The quadrant and sector used were both practically the same as those used by Boscovich. The sector is now in the Turin Academy and has been inspected by Sir Geo. B. Airy, who says "The tube containing the object glass appears to be firmly attached to the bar, and the object glass, though not quite tight in its cell, is tight enough to prevent accidental disarrangement. This tube is unconnected with the rest of the tube. The eye piece also appears to be firmly connected with the bar. The object glass tube projects about two inches beyond the last of its two supports and the eye piece four inches beyond the last of its supports."

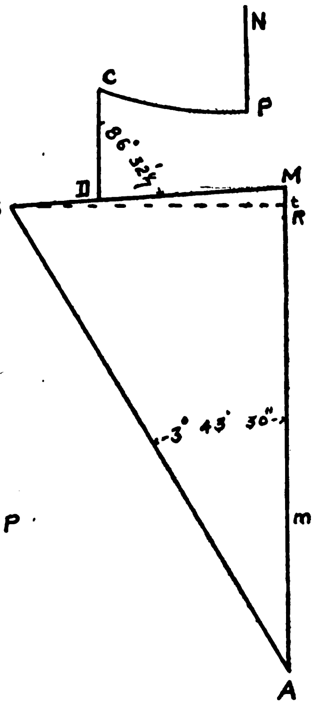
He suggests that the weight might have twisted the eye piece, or that the division might be wrong, for the results of the work were poor, as we shall presently find. The angles were reduced in a manner similar to that described in the previous methods, and the meridian distance, between Mondovi on the south and Andrate on the north, was 64,899.6 toises.

Astronomic work. The star observations seem to have been well made as there is only 2″ difference in the observations. The resulting amplitude was 1° 07′ 44.7″ giving 1 degree equal to 57,468 toises. This result is much larger than other results, and it has been suggested that since the two terminal points were at the beginning of the mountain system, that some error is due to plumb line deflection. His work was revised by Von Zach, who obtained an amplitude of 1° 07′ 44.3″; also Plana and Carlini, about fifty years ago made another measurement giving the amplitude as 1° 17′ 31.07″, which made 1 degree equal to 57,625 toises, a number yet larger in error. Beccaria's work has been universally rejected.

THE NORTH AMERICAN ARC.

1763 In ~~1673~~ two English astronomers, Charles Dixon, and Jeremiah Mason, were employed by William Penn and Lord Baltimore to survey out and locate the boundaries between their respective colonies, namely, between Pennsylvania and Maryland granted to William Penn, and Delaware granted to Lord Baltimore. During the progress of the work the attention of these men was called to the remarkable facilities for measuring a long base line, as afforded by the long boundary line between Delaware and Maryland. This line ran nearly north and south and was over practically level country. So impressed were they with the splendid opportunities, that they wrote and submitted plans to the Royal Society of London, offering to carry out the work if the Society would bear the expense. This the Society agreed to do, furnishing them with the proper equipment, and giving instructions as to how the base should be measured. The equipments sent were the measuring rods called, as we shall see later, levels, thermometers, and a 5-foot brass standard with which the measuring rods were to be compared; these with the instruments already possessed by them, served to perform the work.

The work really required of them was to remeasure with precision the lines already surveyed and run out. The accompanying sketch shows what had been accomplished



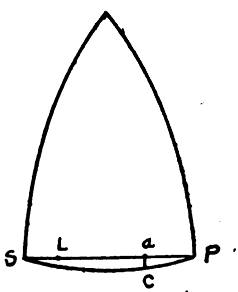


Fig. 2

Fig. 1

in running out the boundaries. NP is a true meridian, PC an arc of a parallel, CD a portion of a true meridian, MDB a straight line making an angle of 86° $32\frac{1}{7}'$ with CD, and BA a line connecting with the terminal point A and making an angle of 3° 43′ 30″ with the direction of the true meridian.

A brief outline of the methods employed in running out the lines will be given, for the accuracy of the length of the degree depends upon the location of certain points, and the approach of AB to a straight line. The measurements seem to have been made in the order NP, PC, CD, AB and BD. The direction of the meridian lines in all cases, was determined by a sector from transit observations of stars. Having regulated their clock they observed a star at the computed time of meridian passage, and made a terrestrial mark. Numerous observations enabled them to establish the line with accuracy.

Instruments. Concerning the sector used little description is given, except that it was six feet in radius and was the first that ever had a plumb line passing over and bisecting a point at the center of the instrument. It was made by John Bird, of London, and belonged to William Penn. Having obtained the direction of their lines, these were run out with a transit instrument. This is one of the first occasions on which such an instrument was used. This instrument, also made by Mr. Bird, consisted of a telescope mounted upon a horizontal axis, the ends of which fitted in the angles of two vertical supports rising from a horizontal bar that was firmly fixed to a vertical axis. The instrument was leveled by means of a spirit level hung on the ends of the horizontal axis of the telescope. A brass frame carrying the vertical axis was provided with a screw, thus enabling the instrument to be firmly set in a post on line. The telescope was of twenty-five magnifying power. For alignment, concentric black and white circles were painted upon both sides of a board 14 inches square. This board moved in grooves between two posts placed each side of the line and when properly aligned, was firmly wedged in position. Below the painted targets a stake was driven in the ground, and a notch cut in the top directly on line, a plumb line being used to effect this.

The instrument itself was placed on a post, usually about 3 or 4 feet along the line from the last established point, and brought into position by sighting on the last two established points, and the line then prolonged. In making this backsight one person was always at the most distant point to make sure that the target had not moved with respect to the hub in the ground. During this process great care was taken to see that the instrument was in good adjustment, the level carefully watched, and the telescope taken out and reversed in its bearings. Usually at least three alignment marks were in sight and served as a check. The measuring was first done with a chain of 66 feet standardized from a brass statute yard in England. This working chain was compared frequently with another standardized chain kept expressly for that purpose.

In running out the parallel CP, the position of P was first ascertained from star observations, and from the meridian line NP an angle, computed 89° 55′ 51″, was laid off. This was the azimuth of a great circle passing through P and would intersect the parallel at a point L, 10′ 00″ west in longitude. It was intended to measure only to the point L; actually, however, they measured on the line prolonged to S, a point 10′ 45″ west of P. This distance was measured and the point marked according to the computed distance PS. The point C was to be 2′ 37″ of a great circle west of P and by computation would be south of a point *a* by 14.1 feet. As they had agreed, however, to run the parallel by sector observations alone, they determined the position of S astronomically, and concluded that as marked from running out PS it was 129 feet north of the parallel. Then computation gave the position of C as 45.5 feet south of *a* and this distance was used to establish the point C. There is then a question of 31.4 feet in the location of this point; however, the authors say that it is unnecessary to discuss which is the better method since the results agreed so closely.

The line CD was run in the same manner as NP. The line AB was started from A, probably with the intention of arriving at D, as later, offsets were made and the line re-run. The angle of AB with the meridian was obtained with the sector, the meridian mark being established as before, six observations agreeing within 3″ at a distance of one mile. The angle as obtained by noting the passage of a star across the vertical of A*p* (*p* being on AB) was 3° 43′ 25″; the same angle was also obtained from the passage of a different star. As a check, from a point *m*, on AM the perpendicular distance to *p* was measured giving as a mean of two measurements 5 chains 14 feet $\frac{8}{10}$ inches; since the distance A*p* was 80 chains, this gave as the angle, 3° 43′ 40″. They adopted as the angle the mean of the three, 3° 43′ 30″.

Base measurement. In February 1768, they commenced their measurements with the rods sent from England. These rods were unlike anything previously used and must have been very cumbersome affairs, judging from their description. No drawings are given, and the following description is that given by the authors (49, p. 312) "The levels used were each 20 feet in length, 4 feet in height, made of pine in the form of a rectangle. The bottom board was 7½ inches in width, top board 3 inches wide and end boards 4 inches wide. The bottom and top were firmly strengthened with boards firmly fixed to them at right angles. The joints were secured with plates of iron and the ends were plated with brass. The plumb line used in setting them level was 3 feet 2 inches in length and hung in the middle of the levels, being secured in a tube from the wind, in the manner of carpenters' levels, on which account we called them by the same name. When the plumb line bisected a point at the bottom the ends were perpendicular. Where the ground was not horizontal, or there were logs etc. to pass over, one end of the level was raised by a winch and pully. The level being set, a short staff was drove into the ground (very near and opposite the plumb-line), in the top of which moved a thin plate of iron, about 12 inches long; at the ends of which were points, which were directed to the intersections of lines drawn on the board that covered the plumb line. By bringing the points in a line with one of the said intersections, if one level was by accident moved, it might be discovered, and brought again to its place. A level being thus marked, the end of the other was brought in contract with it and marked in the same manner before the first was moved; the first was then taken up and set before the last. And so the operation was continued."

These frames or levels must have been held up by some sort of trestles, although no mention is made of them. Mr. Dixon attended to one level, plumb line, and staff; Mr. Mason to the other. The alignment was effected by sighting along the line previously cut out, which being for the most part in woods afforded a long and convenient sight.

During the progress of the work frequent comparisons of the length of the two levels were made by means of the brass standard, 5 feet in length. In order to facilitate their comparison, pieces of brass were fixed into the bottom boards of the level and on each of these a faint line was drawn. At one end of the brass standard, $\frac{1}{10}$ of an inch had been still further divided into hundredths; by these means and the help of a magnifying glass, they were able to obtain the difference between the two levels joined together and eight times the length of the brass standard.

Thermometer readings were also taken that allowance might be made for the change in the length of the brass standard. They adopted as the rate of expansion of brass $\frac{232}{100000}$ of an inch as the variation of a length of 1 foot for a change of 180° Fah.. This amounted to a change of 0.00258″ in the length of the level for a change of 1° Fah..

The following in a specimen form of the notes kept.

"Began at the point N, to remeasure the lines with two rectangular levels each 20 feet in length.

1768	1	2	3	4	5	6	7
February	h						
23	10	M	52	+.12	203	+12.18	−4.97
	3	A	53	+.12			
	The breadth of Brandiwine the 1st. time				8.18	+0.49	−0.47
	Do a 2nd. time where we crossed				12.20	+0.73	
24	Do a 3rd. time				9.87	+0.39	−0.20
	9	M	54	+.08			
	1½	A	44	+.12	325	+16.25	−13.41
	5		39	+.32			

The meaning of the different columns is (49, p. 313) "1 and 2 columns contain the time of day, M signifying morning, A afternoon, 3 the height of the thermometer at Do, 4 the quantity, in hundredths parts of an inch, that the two levels taken together were more or less than eight times the brass standard or 40 feet; 5, the number of levels measured between the times that the levels themselves were measured with the brass standard; 6, the corrections, or quantity in inches, to be added to, or subtracted from, the number of levels measured each day, arising from the levels being more or less than the brass standard; 7, the correction, or quantity arising from the thermometer in inches."

The line NP was completed March 10th, and gave as a result 78,290.72 feet. Line CB was measured March 11th to March 19th, giving 26,608.06 feet. Line BA was measured between March 24th and June 6th, giving a length of 434,011.64 feet. The line NP passed over a few hills, line CD had four small ascents and descents, while line BA had only one hill of gradually rising slope, the remainder being a level plain. At places along BA they encountered swamps and marshes and were obliged to work in water two feet in depth and at one place four feet deep.

The distances across the river were obtained by measuring a base and taking angles with a Hadley quadrant, from which the distances were computed. As a check upon the number of levels, a rope 12 levels in length was stretched alongside, and a tally kept of the number used. The mile posts previously set also served as a check, and in only one instance was there a difference between the mile posts and the levels other than the few feet that would naturally occur.

Astronomic work. The amplitude of the arc was obtained from the difference of observed zenith distances of stars. Observations were made at A, in October 1766, on ten different stars. Half of the observations were made with the plane of the sector east on certain nights, the other half with the plane of the sector west on different nights. Observations had been made at N early in 1764, but it was thought best, on account of the difference in time, nearly three years, to repeat them, which was done in December 1766. The zenith distances here combined with those at A, gave, as a mean from six stars, the amplitude 1° 28′ 44.99″. The latitude of N as determined in January 1764, was 39° 56′ 18.9″; subtracting the amplitude of the arc gave them the latitude of A as 38° 27′ 34″, hence the mean latitude was 39° 11′ 56″.

To find the meridian distance between the parallel of N and A we have—

NP	= 78290.7	feet
CD	= 26608.0	"
Dg = (22 ch.51 l.) (sin. 3° 27 $\frac{6}{7}$′)	= 89.7	"
AR	= 433078.8	"
	538067.	"

In obtaining AR, B*t* was first computed, *t* being a point on AR produced where a great circle, passing through B and perpendicular to the meridian, would intersect it. In computing A*t*, the triangle AB*t* was considered as a plane right triangle, and from A*t* was subtracted *t*R, 15.8 feet, the amount necessary to reduce the point t to a parallel through D. Calling the amplitude 1° 28′ 45″ to have even seconds, they obtained, as the value of 1 degree, 363,763 English feet. Assuming that 114 English feet equal 107 French feet, the result in French feet would be 341,427 or 56,904½ toises at a mean latitude 39° 12′ (approximately).

The above is based on the brass standard of the instrument made by Mr. Bird. This standard was 1-1000 of an inch shorter in a length of three feet than the standard of the Royal Society. Thus the length of the degree measurement should be diminished by 1-36000 of itself, or

10.0 feet, to reduce to the Royal Society's standard. The field standard agreeing correctly with that of Mr. Bird's on being sent to America was found to be 1-1000 of an inch shorter on its return, probably due to abrasion as a result of its constant use. Taking then the mean 1-2000- of an inch, there will be a still further correction of 1-120,000 of the length of the base, or 3 feet, due to the wearing away of the standard, or 13 feet in all. As has been previously stated, there was a question of 31.4 feet in the location of C, which would be about 21 feet for the length of a degree, and this would be added. Since these two tended to balance each other, the result was considered as that first given.

The Astronomer Royal, Rev. Mr. Maskelyne, in reviewing the work, decided to apply the corrections just given, namely

reduction to Royal Society's standard	−10.0 feet
correction due to abrasion	−3.0 "
" " " position of C	+21.0 "

giving thus 363,763 feet 8 equal to 363,771 feet. Also a new comparison had been made between the Royal Society's standard yard and the French foot, giving 76.734 English inches equal to 72 French inches. Thus the result expressed in French unit was 1 degree equals 341,328 French feet or 56,888 Paris toises. The last result is probably the better of the two since the correction should be made, for undoubtedly their azimuth work was more reliable than their latitude. It is to be noted that no reduction of this was made to sea level, which, however, would make very slight difference. This was the longest arc ever directly measured.

PARIS AND GREENWICH CONNECTED, AND THE TRIGONOMETRICAL SURVEY OF GREAT BRITAIN.

In 1783, as a result of a communication from Cassini de Thury, Royal Astronomer of France, England joined with France in an effort to determine accurately the relative situations of the observatories of Paris and Greenwich. This work was the beginning of what developed into a trigonometrical survey of Great Britain and Ireland, and was the starting point of modern precise geodesy. During the progress of the work, which extended over some sixty years, six principal bases were measured.

They are, in the order of their measurements, Hounslow Heath, Romney Marsh, Salisbury Plains, Misterton Carr, Rhuddlam Marsh, and Lough Foyle.

Hounslow Heath Base. This base, measured in June, 1784, was situated upon a fairly level plain. It was about five miles long, the difference in elevation between the two extreme ends being 31.3 feet. It was first cleared of bushes and roughly aligned, and later was accurately aligned and staked out, a transit being used in the work. The terminal marks were made by inserting in the hub of a coach wheel a wooden pipe one foot in diameter and six feet long, the pipe being firmly braced to the wheel and set approximately at right angles to the plane of the wheel. The whole was buried in the ground, the pipe being set in a vertical position, and coming even with the ground. In the top of the pipe a hole, four inches in diameter and two feet deep, was bored. An inverted, truncated brass cone was used to obtain the exact center in measuring. This cone had a mean diameter of four inches, height of five inches, and a very slight inclination to its sides. A lid to the cone had marked at its center two lines crossing at right angles. The cone and its lid being placed in an opening at the top of the pipe served to fix the terminal point. A second lid to the cone, having a small hole through which a plumb-line could be passed, was used when angles were observed. The base was divided into three sections, and the sections were still further divided into what were called hypothenuses of 600 feet; and at these points square pickets were driven into the ground and numbered. A rough measurement was made in June 1784 by the use of a specially constructed steel tape 100 feet long, made by Ramsden.

The tape was made up in links which were practically a foot in length. Each link consisted of three parts; a long plate, and two short ones each of half the thickness of the former. At the extremities of the short pieces holes were drilled, and by means of cast steel pins, the links were joined together. Handles were provided at the ends, from the extremities of which semi-circular portions were cut out in order that circular steel rods, used for marking each hundred foot point, might enter. Additions to the ends were made later in order that the tape might be used more easily when supported.

In the measurement a rope, 600 feet long, was stretched tightly along the line by means of a reel, and the measurements were made with the steel chain beside this rope. The zero end being held over the terminal mark, an arrow was placed as vertical as possible, just touching the semi-circular opening at the end of the handle. The tape does not seem to have been supported, except occasionally where it was held up at the center to clear the ground. In this way they proceeded, a stake being left in the ground every 600 feet. The distances thus marked off were, as previously stated, the hypothenusal distances, and their extremities were the points upon which the levels were taken.

A forward and backward measurement of the first section gave measurements agreeing within 5 inches in 7800 feet. In making the measurement of the middle section a staff, acting as a lever, was used at the rear end of the tape. Two measurements of this section differed 1½ inches in 7800 feet. The third section was measured but once, and its length was found to be 11,804.55 feet. The inclined measurement, corrected for temperature, was 27,408.22 feet. No reduction was made to the horizontal. This measurement really served to establish the 600 foot points for the more precise work to be done with measuring rods and tubes.

Wooden measuring rods. The wooden measuring rods were of riga red wood, external length of 20 feet and 3 inches, the ends being tipped with brass. They were about 1¼ feet wide and 2 inches thick. Four rods were made, one to be used as a standard, the other three to measure with. These latter, being trussed both laterally and vertically, were fairly rigid against flexure. The rods were made of the above given length in order that they might be used either in direct end to end contact, or by the coincidence of graduated lines. To adapt them to the latter method of measurement, narrow pieces of ivory were placed upon the tops at the extremities, and upon these pieces were drawn fine lines, which were 20 feet apart. Extreme care was taken in making these of the precise length given.

In measuring, stands were used every 20 feet, the stands at the 600 foot points being fixed and at the same heights above the pegs in the ground. The other stands were adjustable and brought to proper grade by a screw motion, the grade having been previously established by setting a vertical staff having a horizontal arm which could be slid up and down. The bottom of the arm was set at grade;

hence, when setting up the table, the top had simply to be elevated until it touched the bottom of the horizontal arm.

It was first intended to make the measure by the coincidence of the 20 foot lines; but this method was abandoned after a few lengths had been measured because of the time consumed. The remainder was measured, by end to end contact of the rods. The tops of the tables were arranged so as to hold the rods firmly, and provision was also made for easy longitudinal motion. For preserving the point at the end of the day's work, a plumb-line was suspended from the end of the rod, the bob being held in the brass cup previously mentioned, which was sunk in the ground and filled with water. This served to give steadiness to the plumb-line. A small tripod was firmly fixed near the plumb-line. In the top of this tripod was an adjustable scale, which permitted of both longitudinal and lateral motion. This was brought so its length coincided with the general direction of the line, and a transverse mark opposite the plumb-line. A sentinel placed near served to guard it against disturbance of any sort during the night.

During the progress of the work frequent temperature readings were taken, and the rods were compared with the standard at the begining and end of the day's work. The behavior of the rods during the work was unsatisfactory, owing to their continual change in length. They had not been painted or varnished, and were thus susceptible to the humidity of the atmosphere.

The measurement was made between June 16th and August 3d, considerable time being lost because of rainy weather. The result of one measurement gave as the distance along the hypothenuses of the section, 27,404.31 feet. Allowing for the change in the length as best they could from experiments made later, they decided that 2.02

feet should be added, and from this should be subtracted .07, the reduction to the horizontal. There results, as the base measurement by rods, 27,406.26 feet, at a temperature 63 Fah. and at an elevation of the lower base terminal, Hampton Poor House.

Measurement by glass rods. Because of the variations in the lengths of the wooden measuring rods, and the uncertainty of the corrections, the reliability of the measured length was questioned and it was decided to make another measurement in August, 1784, with the use of a different substance for rods. Glass was selected as the material, and tubes were made twenty feet long and about one inch in diameter. These rods were encased in wooden boxes, 8 inches deep, 8 inches wide at the middle, and 2½ inches at the ends, the boxes being provided with covers. The rods were supported in the box at five points; of which two were at the ends, and the other three were at equal intervals between. They were firmly fastened to the box at the center, and they projected about an inch at each end.

The ends of the rods were ground smooth and at right angles to the bore. One end was called the fixed, the other the movable. In the fixed end a hollow cork, three inches long, was inserted; inside this cork was a hollow cylindrical tube open at the outer end but with a hole tapped out at the other end. A steel pin with a screw at one end and a head of the size of the glass tube was inserted, which completely closed the tube and presented a terminal metal cap, whose surface was at right angles to the axis of the tube. The movable end was arranged in a similar manner, except that the steel pin did not screw through the metal cylinder but pressed against a spiral spring at the end. This metal pin carried a scale, and terminated not in a plane surface like the other but in a spherical surface. On the exterior of the glass tube was a scratch so placed that when the movable end was pressed in, a mark on the scale coincided with the mark on the glass. The distance between the surfaces at the fixed and movable ends was 20 feet. Three such rods, properly encased, were made. As a further precaution against temperature changes, the cases were covered with linen glued to the wood. Two openings in the boxes, one near each end, permitted the reading of thermometers inside the cases.

Rollers were provided at each end of the cases, so arranged that they carried the weight or were released from it at the will of the operator. The measurements were made in precisely the same way as with the wooden rods, the bars being supported at the terminals every twenty feet. For ease in measuring, upright pins were fixed in the adjacent ends of the case, and these were connected by a rod having a slow motion screw. By means of these, the abutting ends of the glass rods were pressed together until the scale mark of the movable end coincided with the mark on the glass.

The resulting length of the base, measured inclined, was,	27402.8204	feet
The following corrections were applied:		
Reduction to horizontal	−0.0714	"
Temperature correction of rods	{ +0.3489	"
	{ +1.6946	"
Reduction to sea level, elevation of 54 feet	−0.0706	"
Length at sea level	27404.7219	"
Temperature correction made later	0.7082	"
Final result	27404.0137	

In 1787 the actual work of triangulation, designed to connect the Royal Observatories of Greenwich and Paris, was commenced. The delay had been caused by the lack of a suitable instrument, but in July, 1787, Ramsden finished and turned over to them what has been known since as the Great Theodolite. This was a remarkable instrument, being much larger than any ever before constructed, and was a great improvement over their former quadrants for angle measure. Much of the angle work in Great Britain and Ireland during the following sixty years was done with it. Very few changes in the construction of it were made, except in the providing of additional microscopes, and according to Col. Clarke, it was apparently as good at the close of the work as when it came from the makers.

This instrument consisted of three parts, irrespective of the stand on which it was supported. The lower part consisted of the leveling screws, the vertical axis, and the microscopes for reading the horizontal circle. The part above was made up of the horizontal circle, the hollow vertical axis fitting over the other, and the Ys for carrying the telescope. The top part had the telescope, its horizontal axis, and the vertical circles attached to it. Originally two microscopes were provided, but others were added later.

The horizontal circle was three feet in diameter and graduated to ten minutes, the micrometer microscopes reading to single seconds. Across the top of the vertical axis and placed horizontally was a flat bar extending one foot each side. From this rose the Ys which supported the axis of the telescope. This horizontal bar was trussed from the vertical axis in order to give it greater rigidity. The telescope had a focal length of 3 feet, with a 2.5 inch aperture and magnifying power of 54. Two vertical circles attached to its horizontal axis permitted of its use as an astronomical instrument. Originally a second telescope of the same size as the other was placed below the horizontal circle, but this was soon discarded. Provision was made for centering over the station. The weight of the instrument was 200 pounds. A second instrument, differing only in details of graduation and number of microscopes was made a few years later, and also an 18 inch theodolite similar in construction.

One other instrument deserves mention, the Troughton and Simms's 2 foot theodolite. This was a repeating instrument. The telescope of 2 feet 3 inches focal length, $2\frac{1}{8}$ inches aperture, with its two vertical circles, was supported by two tall pillars rising from a drum. To this drum were attached six microscopes for reading the two foot horizontal circle at the base of the drum, which was graduated to every 5 minutes of arc. Two levels on the top of the drum served for leveling. The whole weight of drum, telescope, etc., rested on friction rollers.

During the process of the work the instruments were sheltered, and on good foundation. The angles were observed by the direction method; that is, starting from any one object at any reading, they pointed to the next station and read the angle, and so on around to the first, then reversed the telescope and repeated the pointings in the opposite direction. This constituted a set. The circle was then shifted and the process repeated. Azimuths, when obtained, were from Polaris at elongation.

Triangulation. The work of triangulation was commenced in August, and in September, 1787, Gen. Roy met the members of the French Academy, Messrs. Cassini de Thury, Mechain and Legendre, to plan for the connection across the English Channel. The work was not completed until the following year. Many of the angles were taken at night by pointing to lights provided with reflectors, which had been erected at the different stations. Elsewhere the signals were either spires or specially constructed poles erected and braced.

A verification base was measured at Romney Marsh. The line was very carefully aligned out and posts driven every 20 feet. The top of the 100 foot post carried a graduated scale that, by the use of a slow motion screw, could be moved longitudinally in the direction of the line. Near the 100 foot posts were placed two other posts, one on each side. Resting on each post were shallow troughs, or coffers as they were called, each being 20 feet long, except the two end coffers terminating on the posts next to the 100 foot posts. These were 19 feet and 4 inches long. In measuring, the tape, the same which had been used at Hounslow Heath, was laid flat, one end being held firmly at the zero mark by a connection with the auxiliary post, while from the other end at the auxiliary post was suspended a weight of 28 pounds connected to the tape and passing over a pulley. In this way a constant tension was administered, the 100 foot point being brought over its proper post and its position marked by sliding the graduated scale into coincidence. The mark thus served to denote the end of the 100 foot point and the beginning of the next. Usually a distance of 300 feet was measured, there being coffers enough for this distance. After one section had been completed they were taken up, relaid and the work carried on as before. The terminals were marked by iron pipes driven in the ground.

The length measured between October 15th and December 4th, 1787, gave 28,536 feet, 8.835 inches. This corrected for temperature, slope of line, and wear of tape, became 28,532 feet, 11.202 inches. Reducing to sea level by subtracting 0.166 inches gave 28,532 feet, 11.036 inches or 28,532.92 feet, the length at sea level and temperature of 62° Fah., which was the temperature adopted in computing the length of the Hounslow Heath base. The computed length coming from the Hounslow Heath base agreed within 4½ inches.

The triangulation, carried into the north of France with the values of the angles as furnished by Cassini de Thury, checked up on the base at Dunkirk within 7 feet, being too large by this amount. This base however had been measured with wooden rods (see pp. 34) and was probably not precise. By taking the length of the same base as computed from triangulation starting from a base at Paris, the agreement would be within 1 foot, 3 inches. From the best judged combinations, Gen. Roy obtained the value of one degree of the arc between the parallels of Paris and Greenwich as 60,840 fathoms or 365,040 feet, for a mean latitude of 50° 9′ 27″. The amplitude of this arc was 2° 38′ 36″.

Having once started upon geodetical work, it was perfectly natural that the work should be continued. This was done, and plans were made which resulted in a trigonometrical survey of Great Britain and Ireland during the succeeding years. In 1791 the Hounslow Heath base was remeasured, this time with steel chains. Two new chains made by Ramsden were used. They were practically like those previously used, except that instead of having 100 links, these had 40 links, each 2½ feet long. Brass handles were provided at each end which, being flat on the bottom, would slide easily over the scale placed upon the 100 foot posts. Arrows were drawn on the handles to denote the length of the chain.

The tape was supported throughout its entire length by deal coffers arranged on trestles after the manner of the Romney Marsh base, much the same apparatus as was then used being again used here, with identically the same methods. The weight used for stretching the tape was probably 56 pounds. In the Ordinance Survey Report (76, pp. 209) and in Clark's Geodesy (15, pp. 16), the weight given is 28 pounds. But during the experiments made before and after the measurement, in order to determine the length of the tape and its expansion, the weight of 56 pounds was used; and in applying corrections to obtain the true length of the measured line, no attention was paid to elongation, as should have been done if the tension used in measuring was other than that used in standardizing.

Five thermometers were laid alongside the tape and allowed to remain there until all showed practically the same temperature. This usually took from 7 to 15 minutes. When the sun shone the chain was covered with a white cloth, the ends being put over the first and last coffers to prevent the circulation of air.

The result of their inclined measurement was a base line of	27401.755	feet
To this the following corrections were applied:		
Correction for excess in length of chain and half the wear	+ 2.0539	"
Correction due to difference in temperature above that at which the tape was standardized	+ 2.8519	"
Correction to reduce to temperature of 62° Fah.	— 2.2596	"
Reduction to horizontal	— 0.0857	"

Resulting length at 62° Fah.	27404.3155	feet
To this should be applied the reduction to sea level (omitted by them)	— 0.0706	"
Hence the result at sea level was	27404.24	"

Comparing this with the measurement given by the glass rods (see p. 139) it is .23 feet greater, a difference of a little less than three inches. At this time the wooden pipes used as terminals were replaced by cannon set muzzle upward, the center of the bore being the terminal mark.

Verification base on Salisbury Plains. The base was measured in 1794 by Lieut. Col. Williams and Capt. Mudge. The location of the base was not as level as that at Hounslow Heath, it being rather irregular. The same apparatus, tapes, trestles, coffers, etc., used last at Hounslow Heath were again used here. In obtaining the inclination, however, instead of direct leveling, the angle of the slope was measured by a small transit. Before measuring, the chains were examined and found to have the same relative length as when examined at the close of the Hounslow Heath work. They were also measured at the close and allowance made for wear. The result reduced to the level of Beacon Hill, one extremity, all corrections being made, was 36,575.401 feet. This, when reduced for the elevation of 669.5 feet, became 36,574.232 feet the length at sea level. The terminal points were marked with cannon after the manner of those at Hounslow Heath.

THE ARC IN ENGLAND.

The results of the Trigonometrical Survey up to 1802 gave data for the establishment of an arc from Dunmore on the south, to Clifton on the north. For checking the extension of the triangulation northward, a base was measured by Major Mudge at Misterton Carr, in 1801. This was done in June and July, with the same apparatus in all its details as was used at Salisbury Plains and Hounslow Heath. In addition a chain of similar construction but fifty feet in length was occasionally used.

After all corrections had been made, the result, reduced to sea level and the standard temperature of 62°, was 26,342.19 feet. The ends of this base were marked by two blocks of oak having square holes in the top, which were filled with lead smoothed off even. Upon the surface of these, intersecting diagonal lines were drawn. The amplitude of the arc was obtained from the difference of zenith distances of stars observed at Dunmore and Clifton, from May to August 1802. The mean from 17 stars observed on different nights gave as the amplitude 2° 50′ 23.38″. The triangulation distance was 1,036,337.0 feet, hence the result gave 60,766 fathoms as the value of 1° at a mean latitude of 52° 50′ 29.8″.

In connection with further triangulation work, two other base lines were measured with the same apparatus; one at Rhuddlam Marsh in 1806, which gave as a result 24,514.26 feet at sea level and referred to the standard brass scale; the other at Belhelvie Sands in 1817, which gave 26,515.65, feet referred to the same standard. In the following years, when the survey was extended to Ireland, a base was measured at Lough Foyle. At this time the principle of compensating base apparatus was in vogue.

The base apparatus was designed by Major General Colby, the designer's idea being that if the bars were made of different metals, they could be so arranged that they would preserve an unvariable distance between two points. The following is a description (76. pp. 201-202).

"The compensation bar consists of two bars, each 10 feet 1.5 inches long, 0.5 inches broad, and 1.5 inches deep, placed 1.125 inches apart and firmly connected at their centers by two small transverse steel cylinders, not quite in contact. At each extremity is a metal tongue so connected by pivots to the bars as to admit freely of expansion and yet to be quite immovable otherwise. These tongues are each 6.2 inches long and on a silver pin at the extremity of each is marked the compensation point." This compound bar was placed in a deal box, and was kept from moving lengthwise by means of a brass stay firmly to the bottom of the box at the center and projecting upwards between the two small steel cylinders before mentioned. A long level for leveling the bars was fixed to the upper surface of the brass bar and was read by means of a glass covered opening in the top of the box. The tongues carrying the compensation points projected beyond the box but were carefully protected. These points, it is to be noted, and not the bars themselves, lay in the line of measurement. There were six of these designated as A, B, C, D, E, G, respectively.

In measuring, the boxes were supported on stands at the quarter and three quarters points. These supports provided for both lateral and longitudinal motion. The distance between the compensating points on adjacent bars was made exactly six inches by means of a compound microscope, which consisted of two microscopes, one of brass and the other of iron, so placed that their foci remained six inches apart. Hence in the measure, one microscope was placed over one terminal point of the bar and the other terminal point brought under the second microscope. Seven microscopes were used.

For transference to the ground a plate, which could be set in the ground and which carried a vertical cylinder, was used. In the top of this cylinder a movable disc containing a fixed point could be placed under the microscope This served as the terminal of one day's work and the beginning of the next.

The base was measured by parts during 1827 and 1828, and gave as a result 41,640.8873 feet, when reduced to sea level and expressed in terms of the ordnance standard O_1 at a temperature of 62° Fah. In 1849, the Salisbury Plain was also remeasured with this same apparatus, and this base was found to be 36,577.8581 feet at sea level and referred to the ordnance standard O_1 at 62° Fah.

Zenith sectors. The Ramsden zenith sector used in the astronomic observations was, compared with our modern instruments, rather a ponderous affair. It consisted of two parts, an inner and an outer frame. The outer frame supported the whole apparatus, the sector tube and its adjuncts. It was about 12 feet high and in the form of an open truncated pyramid, the base being 6 feet square and the top one half this. Inside this was another hollow frame suspended from the top and supported at the bottom by a conical surface resting in a metal cavity. This inside frame supported the sector, the main telescope of which was 8 feet long with an object glass 4 inches in diameter. The extent of arc was 15½° The instrument was read by means of a plumb-line and was provided with facilities for placing it vertical and reversing its position. This instrument gave good results but it was very cumbersome to move from one station to another and was superseded later by the Airy zenith sector.

This latter sector was like the Ramsden in general outline but differed in details. It could be more easily reversed, a level was substituted for the plumb-line, and the instrument was, as far as possible, cast in one piece. The instrument was in three parts, (76, pp. 757) "The framework, the revolving frame, and the telescope frame.

"The framework is cast in four pieces; the lower part, an inverted rectangular tray with leveling footscrews; two uprights, with broad bearing pieces, very firmly screwed to the inverted tray; and a cross bar uniting the tops of these uprights, whose ends are cut as screws. Through the center of this bar passes downwards a screw with a conical point, which together with the vertex of a cone rising from the center of the inverted rectangular tray, determines the axis of revolution, and forms the bearings of the revolving frame.

"The revolving frame is cast of gun metal, in one piece. It is also in the form of a tray, strongly ribbed at the back, having four lappets or ears acting as stops in the revolution. In the center of this frame is a raised ring of about nine inches diameter, forming the bearing plate of the telescope frame. Concentric with this ring at each end of the frame are the divided limbs, which have a radius of 20.5 inches, and are divided on silver to every five minutes; the divisions are numbered from 0° to 360°, interrupted by the portions of the circle which are wanting. There is also at each end a raised clamping-limb, roughly divided, to which the clamp for securing the telescope frame at the required zenith distance is attached; the graduation furnishing zenith distances on both sides of the zenith, and also circle readings corresponding with those of the divisions of the limb, the pointer-reading being given by a small index attached to the clamp. On the reverse side of the revolving frame are mounted three levels, the divisions of which are numbered from right to left.

"The telescope frame revolves in a vertical plane by a horizontal axis or pivot, of 3 inches diameter, passing through a corresponding cylindrical hole in the revolving frame. Cast in one piece with the telescope frame, are the ring for holding the object-glass-cell of the telescope, the four micrometer microscopes, which are afterwards bored through the metal, and the eye-piece. The micrometers are of the usual construction, the wires intersect in an acute angle and have a range of about 10 minutes on the divided limb. The value of a division of the micrometers reading the limb is approximately a quarter of a second.

"In the eye-piece of the telescope are five meridional wires carried by a fixed plate, and a single wire at right angles to them moved by a micrometer screw. The focal length of the telescope is 46 inches, the diameter of the object-glass 3.75 inches, and the magnifying power usually employed about 70.

"The revolving frame is generally reversed in its position with regard to the pivots, once at each station, so that the reading of the zenith point as given by the pointer at the lower end of the telescope is for one part of the observation about 12° 30′, and for the other part about 192° 30′."

SVANBERG'S ARC OR THE LAPLAND ARC REMEASURED.

Notwithstanding the apparent accuracy of Maupertuis's work, the results seemed large as compared with those which had been obtained in later years. Certain members of the Royal Academy of Sweden, interested in geodesy, believed that Maupertuis's results had been influenced by the deflection of the plumb-line due to the mountains and, in 1799, Jons Svanberg was sent to Lapland to make investigations. The conclusion that he arrived at was that the deflection would be inappreciable, yet it was felt that while no great error entered, small errors resulting from the severity of the climate, flexure of the sector, possible disarrangement in transportation, etc., might have entered. Accordingly the Royal Society of Sweden sent a petition to the King setting forth the advantages that would accrue from a remeasurement. This was favorably received and the King requested the Royal Academy to undertake the work.

A preliminary trip was made in 1801, Messrs. Svanberg, of the Royal Observatory, and Ofverbom, Chief Engineer of the Bureau of Surveys, being sent. The object of this trip was to make a reconnaissance, select the terminal points and erect observatories, choose the stations and make all researches possible for the stations used by Maupertuis's party. They returned in October and began preparations for the second trip. Two men were added to the party, Messrs. Holmquist, of the University of Upsale, and Palander, of the University of Abo.

This party left in January, 1802, taking with them the standard toise, the meter, and a new repeating circle, made by Lenoir, all recently received from Paris. In Svanberg's published report much space is given to theoretical investigation. Concerning this he is almost painfully minute, but concerning details of instruments and methods used he is very meagre. No description of the instrument is given, but it was evidently used for both the measurement of the horizontal angles and the astronomic work. The recorded angles indicate that the decimal system of graduation was used, a right angle being divided, not in the usual way, but into 100°, each degree into 100′, etc.

This party extended the triangulation of the Academicians both to the north and south, using most of the mountains employed by them. The southern terminal of the

Maupertuis base was found, but no direct evidence of other stations. This was used by Svanberg as the starting point for his base-line.

Base apparatus. The rods used were of iron, cold hammered, a little over 6 meters in length, and having a cross section 24.3 mm. by 31.1 mm. At each extremity the bars were cut away on the sides, thus enabling the end of one to slide by the other. Upon each top was placed a layer of silver having a fine line traced upon it. These lines were six meters apart, and were laid off by means of the standard 2 meter iron rod sent from Paris. The manner of obtaining these 6 meter graduations was as follows. As the standard rod was 2 meters between the external faces, two short additional iron pieces of the same cross section were carefully made and clamped to the extremities of the standard bar, these affording a slight line designating the length which was taken off by beam compasses and used three times to define the 6 meter marks. Four such bars were made, one being used as a standard.

In measuring, the bars were supported at three points in a special frame, these frames resting on trestles about ½ a meter in height. In the tops of the supporting frames, for ease and alignment, were sliding pieces governed by screws permitting a lateral motion and also an arrangement whereby the whole bar might be made to slide along in order to bring the two lines in coincidence.

The measurement was commenced February 22nd, 1802, starting at the southern point, the same one as used by Maupertuis. Owing to the prevailing mists they were unable to see the northern terminal base mark at all times, and alignment was effected by sighting on poles, previously set up on line. Although most of the base was over ice, a small portable sector was used to obtain the inclination of the rods where necessary.

In making the measurements, Messrs. Ofverbom and Holmquist made the alignment and placed the bars approximately. Messrs. Palander and Svanberg measured the inclination to the horizon, read the thermometer, and brought the marks on the bars into coincidence. For leaving a terminal point at the end of a day's work, a plumb-line was not used, owing to the prevailing winds. Instead, a metal bar in the form of an inverted T was employed. Across the horizontal part was placed a small level bubble and normal to the bubble, when in the center of the tube, a fine line was drawn up the vertical part of the T, terminating in a sharp point. When it was desired to bring down the point, snow was first heaped up and closely packed into a hard mound under the end of the measuring rod, and into this pile of snow was driven a stake, having nailed on its top a brass plate marked by a point in its center. In this point was placed the sharp point of the inverted T, the bubble was brought to the center of the tube, and the distance from the vertical line to the scratch on the measuring bar was then measured by means of a short scale graduated to millimeters.

They terminated this base measurement at a fence dividing two country villages. There seems to be no evidence of their finding the northern base terminal of the Academicians. Their own terminal was marked by burying in the earth a charred piece of timber, having fixed on its top a brass plate with intersecting lines upon it, the whole being carefully covered up.

Their resulting measure was	14451.912	meters.
To this should be applied		
Correction for mean temperature of −4.313C	−.7131173	"
" " inclination	−1.4090648	"
Reduction to center of signal at Niemisby	−1.4318	"
" " " " " " "	−.0047	"
" " sea level	−.1102744	"
Final result	14451.116	"
or using the standard toise of Peru	7414.4919	toises.

It is inferred that they made frequent comparisons with their six meter standard previously mentioned, but no direct statement to that effect is made. The length of this base is about 6 toises longer than that as measured by Maupertuis and the Academicians.

Twenty-seven triangles were used in their system of work, each angle being repeated a number of times, 5 being the minimum and 50 the maximum. The majority, however, were measured 30 times. They devoted considerable attention to signal construction and erected good signals. A trunk of a tree 20 feet long was used as a mast; this was braced by four other sticks placed in pyramidal form and cross braced at the bottom. The foot of the mast, which was about 2 metres from the ground, was strongly cross braced to the supporting sticks and the whole held firmly in position by rocks piled around the base. In the top of the mast was a long iron pin serving as a pivot for a rectangular vane. This vane, pivoted at the middle, was open in the center, thus affording a central sky background surrounded by a frame of wood. The outside of the signal at the base was covered with brush or logs up to the height of the foot of the mast. The signal then presented a solid looking base and an open space at the top of the mast, either of which could be used according to the light falling upon it and the position of the observer. Most of the stations were occupied directly, since this was easily done by removing the brush at the foot of the signal.

In obtaining the most probable value of the angle from repetitions, a complicated method of reduction was adopted which amounted to giving weights to the angles. The method used was wrong, for, if all observations were equally well taken, the mean should have been used. The angles were corrected for eccentricity of instrument, reduced to the center of the station where necessary, and to the horizontal. Spherical excess was recognized and allowed for, and the triangles computed according to the Delambre method. The triangulation distance between the parallels of the extreme stations, Mallorn and Pahtavara, was 180,827.68 metres.

During September, azimuth and latitude observations were taken at Mallorn. The latitude observations were made by circum-meridian observations on Polaris at upper culmination on different nights, and gave, when reduced, 65° 31′ 30.265″ as the latitude. The azimuths were determined from observations on the sun. The astronomic work, at Pahtavara, was done in December, using Polaris at upper culmination as before. The resulting latitude was 67° 08′ 49.830″, hence the amplitude of arc was 1° 37′ 19.566″. Dividing 180,827.68 metres by the amplitude in degrees gave 1 degree equal to 111,477.408 metres or 57,196.159 toises.

Airy (6. pp. 210-211) has pointed out that the amplitude should be decreased 0.28″ because of error in calculation of nutation. This would give one degree equal to 111,482.747 metres or 57,198.9 toises. From an appended note in the report, it would appear as though Svanberg was not absolutely sure whether his standard bar had been compared with the standard platinum metre at 0° centegrade or 16¼° C. The computations have been made under the suppositions that they were compared at zero. If they were compared at 16¼° C, the terrestrial distance would have been 180,794.06 metres and 1 degree would equal 111,456.68 metres or 57,185.524 toises. It is to be regretted that the stations of the Academicians were not found in order to know where the two results differed.

FRENCH ARC, DUNKIRK-MOUNTJOUY. USED FOR THE ESTABLISHMENT OF THE LENGTH OF THE METRE.

In 1790, as a result of the interest of scientific men in France to reform, as they believed, the existing system of weights and measures, a committee was appointed by the Royal Academy, at the request of the king, to consider the question. This commission, and others that followed, decided that the standard unit of length should be $\frac{1}{10000000}$ part of a meridian quadrant of the earth. Their object was to establish an invariable standard of measure and length. This, we recognize at the present day, is impossible to do by taking any certain portion of the earth's quadrant as a unit, for probably no two meridian quadrants are absolutely the same. What they accomplished was the establishment of a decimal system with an arbitrary unit as a base, which was practically $\frac{1}{10000000}$ of the earth's meridian quadrant.

Commissions of the Royal Academy recommended that a new meridian arc be measured between Dunkirk in France, on the English Channel, and Barcelona in Spain, on the Mediterranean Sea. This arc would have an amplitude of about 9½°, 6° lying north of the 45° parallel and 3° south. Among the men serving on the various committees of the Royal Academy, and also appointed later by the committee of public instruction, were three who were closely identified with the determination of the unit of length. These three were Messrs. Delambre and Mechain, who superintended the astronomical and geodetical work, and Borda, who constructed the measuring rods and provisional metre. The geodetical work was much delayed owing to the French Revolution. In carrying out this work, Delambre was appointed to the northern and Mechain to the southern part.

At this time two new principles were brought in which have dominated most nations for almost a century, namely, the principle of the repeating circle and that of the metallic thermometer. The repeating circle was used both for the measurement of the terrestrial angles between stations, and also on the astronomic work for determining the zenith distances of stars. In construction, it consisted of a vertical axis supported by a three-foot screw. At the foot of the axis was a horizontal circle with clamps and tangent screws used for orienting the instrument when necessary. From the top of the vertical axis rose a U-shaped piece of metal, the sides of which formed the support for the horizontal axis of rotation of the circles. Perpendicular to this axis of rotation was a short axis to which the reading circles were attached above, their plane being perpendicular to this axis; below, the axis terminated in a circular counterpoise, the height of the U-shaped piece being sufficient to permit its rotation through. The circles, two in number, were placed over each other, and were separated by a groove in which the tangent clamp of the upper telescope, moved. The upper circle was graduated decimally and had a telescope carrying four verniers. A second telescope, mounted eccentrically, was attached to the under circle, which was not graduated.

The lines of sight of the telescopes were parallel to the planes of the circles, and the telescopes were free to move in this direction. The upper circle was graduated decimally into 400 degrees, each right angle having 100 degrees. From the nature of the construction of the axis, the circles could be brought into any desired plane from a horizontal to a vertical. A level bubble attached to the under telescope was used for making the circles vertical ordinarily, except when zenith distances were being measured. In this case the verticality was obtained by means of a plumb-line, which was suspended from a projection on the under telescope and passed over a mark on a like projection, on the lower part of the circle. The motions of the two circles being independent and at the will of the observer, the angles could be easily repeated.

The mode of procedure was first to bring the plane of the circles into the plane of the objects, between which the desired angle was to be taken, and clamp the axis of the circles. The upper telescope was set at zero and brought to point on either of the objects; for instance, the right hand one and then the circle clamped. The lower telescope was then turned to the left hand object and also clamped. The angle between the telescopes was the angle between the objects, but of course it could not be read since there were no graduations on the lower circle. Hence the circles were unclamped and rotated in their plane (the telescopes being undisturbed) until the under telescope was brought to point at the right hand object. By this means, the upper telescope was pointing to the right of the right hand object by the amount of the angle measured. Then by unclamping the upper telescope and pointing it at the left hand object, the reading of the circle gave double the angle between the objects. In this manner, the double angle could be obtained as many times as was desired.

The main object of the repetition circle was to increase the accuracy of the measurement of the angles by the elimination of errors of graduation and eccentricity of the instrument, and the doing away with the reading of a single angle. This method was adopted by nearly all continental countries and has been used in France almost up to the present time. Theoretically the results should be more accurate, and undoubtedly the results at that time were increased in accuracy by its use. At the present time, however, it is rarely used for the most refined work, for with all the advance in the construction of instruments and the use of micrometer microscopes, it is a question whether any advantage would be gained by its use. This method was never used in England.

The use of the metallic thermometer principle in the construction of the base apparatus was a decided advance over the wooden rods used in all previous cases, except recently in England. By the combination of two metals it was hoped to arrive at the true temperature of the measuring rods, by noting their relative expansion. The work in recent years has caused the accuracy of such thermometers to be questioned. Four such measuring rods were made by Borda and were known as rods number I, II, III and IV. Number I was made from the toise of Peru and was exactly equal to that at the temperature 13° R. This rod served as the standard, and from it the national prototype metre was afterwards made and deposited in the archives.

The construction of the measuring rod was first a strip of platinum 2 toises long, 6 lines wide and 1 line thick. This was covered with a strip of copper of the same width but 6 inches shorter. At one end the copper was rigidly attached to the platinum by three screws, the end surfaces of the copper and platinum being in the same vertical plane. The copper was thus free elsewhere to move longitudinally along the platinum by expansion. Under the free end of the copper strip was a vernier attached to the platinum strip, the reading of which indicated the relative expansion of the two metals. At the end of the platinum, which was free and not covered by brass, was a scale sliding in a grooved slot in the strip. This scale was divided into ten thousandths of a toise; and by means of a vernier, marked on one of the edges of the grooves, readings could be taken to hundred thousandths of a toise.

The length of the toise was from the plane of the fixed end to a mark on the free end of the platinum, and the use of this sliding scale was to obtain the distance from this mark on the free end to the fixed end of the adjacent measuring rod. The readings of both the sliding scale and the vernier under the copper rod were obtained by the use of microscopes. The strips of metal being very thin, and therefore flexible, were supported their entire length on boards well dressed, and painted in different colors to distinguish the rods from each other. They were kept in position on the boards by mountings, which could adjust them to a straight line and which were at the same time not sufficiently rigid to prevent expansion.

These rods were free to the air, but were protected from the sun above by a board attached to and parallel with the board on which the rods rested, at about 4 inches above it. This was sufficient protection from the direct rays of the sun except when it was low. In this case a cloth was laid over the frame on the side of the sun, thus preventing its rays from falling directly on the rods. These frames were supported on foot rests which consisted of a wooden top provided with three screws bearing on the metal plate below. The frame of the rods was held in position by these three screws, and since the screws were arranged in triangular form, the rods were between the screws. The metal part of the foot-stools terminated in points which could be driven into the ground. These foot-rests were placed under the measuring rods, at a distance of two and one-half feet from each extremity. For alignment, two iron pins or sights were placed in the top, coming directly over the rule, so that when they were on line the rule was also on line. Also on this covering were two supports, one each side of the center, on which were placed the feet of the level which was used to measure the inclination of the bars.

Two base lines were measured, one at Melun, the other at Perpignan, and in both cases they were broken bases, that is, bases made up of two intersecting lines. The base at Melun differed from a straight line by about 49′, that at Perpignan by about 23′. The angles being obtained by using a repeating circle and combining the results of angles taken at the intersection point to the same signal.

In measuring the bases, they were first aligned by sighting from the intersection to the terminals. At this point a stake, 3 feet long and six inches in diameter, was driven, the top projecting a short distance above ground. At the end of every 100 toises a pin was driven, the correct setting of these pins being obtained by first making a hole in the ground with a wooden hub, and then replacing this hub by the iron pin, very accurately aligned. Several of these pins were established from one position of the instrument, which was then taken up, set over the last pin driven, and more of the line then staked out. The measuring rods were aligned by sighting at these iron pins. The terminal of the first rod was carefully brought over the starting point by plumbing down to the mark in the ground, allowance being made for the thickness of the plumb-line. The adjacent rod was placed on its foot-rests, aligned and brought near enough to the first rod so that the sliding scale could be brought in direct contact with its end. The third rod was similarly placed, the vernier readings of the metallic thermometer and contact scale were read, and the inclination of the bars obtained. The sector was reversed on each bar. The rear rods were then taken up, carried ahead, placed on their foot-rests, aligned, and the process repeated. For terminating a day's work, a straight stake was driven in the ground and a lead plate fastened in its top. The point was determined by plumbing down, and marked by drawing two intersecting lines at right angles. This was covered during the night, care being taken not to disturb it in covering, and then it was used as a starting point for the next morning's work.

Following is a specimen form of the notes kept during the base measurement. The readings, with the exception of those pertaining to the level, are expressed in hundred-thousandths of a toise. The level headings indicate the direction in which the marked end of the level was pointing. The double inclination is the difference between the two level readings. The value dN represents the difference in level between the extremities of the measuring bar and its sliding scale, and is found from $dN=(r-l)\sin I$, where r=rod length, l the scale reading and I the angle of inclination. For a rising slope dN was called positive, and for the reverse, negative.

No. of measuring bar	Metallic thermometer reading	Scale reading	Level		Double inclination	Reduction to horizontal	dN	dN
			Brie	Malvois			+	—
1	416.0	409.8	53.2	66.2	13	9	189	
2	416.5	235.9	81.4	38.0	43.4	101.4		637
3	415.0	346.4	50.4	68.4	18.0	17.3	262	
4	420.4	506.6	72.1	48.1	24.0	30.5		349

In measuring, the lower surface of the sliding scale was considered as the upper surface of the measuring rod, and in consequence should be brought in contact with the upper part of the adjacent rod in order to form a continuous surface. Were this done every time, and the inclination accurately obtained, the sum of $+dN$ and $-dN$ ought to agree with the difference in level between the base terminals. The difference in level obtained from the dN's was 7.364 toises, while the actual difference was 9.0 toises.

The result that Delambre presented to the commission was that the length of the base of Melun, at sea level and in arc, was 6,075.900,069 toises. In obtaining this, all corrections due to reduction of scale reading, metallic thermometer reading, reduction to a straight line, etc., were made.

The base at Perpignan was measured in a similar manner, with the resulting value, obtained by Delambre, of 6,006.247,848 toises. The terminals of the base line were marked by a solidly constructed cut masonry monument, the foundation of which rested on rock. The exact point was marked by a copper cylinder inserted in the monument, the terminal point being the center of several concentric circles drawn on the surface of the cylinder.

This cylinder was situated in a square depression of the monument and was covered with a layer of lead, upon which was placed the fitting pyramidal capstone, projecting a little above the surface of the ground. The ground was then paved and the whole surrounded by cut stone pillars a few feet in height, arranged in circular form.

Astronomical work. Azimuths were observed by Delambre at Watten, Paris and Bourges, and by Mechain at Carcassonne and Montjouy. These observations were made principally upon the sun. Morning and evening observations were preferred, but in this they were not always successful, because of clouds; hence observations were made at different times during the day, or when the sun passed the vertical of some distant signal. At Montjouy numerous observations were made, some being upon Polaris.

Latitudes were made at Dunkerque, Paris, Evaux, Carcassonne and Montjouy. The repeating circle of Borda was used principally. In the calculation of the triangulation, the earth was considered as a spheroid, generated by the revolution of an ellipse about its minor axis. Spherical excess was recognized, the arcs reduced to chords, and the computations made. The meridian distances were obtained by starting with the formula

$$\frac{dA}{dL} = \frac{m\cos I}{(1-\sin^2 I\cos^2 L)^{3/2}},$$ which represented the radius of curvature of the meridian, and developing it into the formula

$$\begin{aligned}\frac{A'-A}{\cos^2 I} = \alpha(L'-L) &- 2\beta\sin(L'-L)\cos(L'+L)\\ &+ 2\gamma\sin 2(L'-L)\cos 2(L'+L)\\ &- 2\delta\sin 3(L'-L)\cos 3(L'+L)\end{aligned}$$

etc., where L' and L represent the latitudes, and α, β, γ and δ are functions of the ellipticity.

A committee appointed to review the work decided on the length of the base of Melun as 6,075.900 toises (or semi-modules as they called them) and the base of Perpignan as 6,006,249 semi-modules. Since the value of one base computed from the value of the other base by means of the triangulation did not exactly check, a question arose as to how the calculations should best be made; whether to compute the work wholly from one base, to use the mean, or to calculate part of the work from one base and the remaining part from the other base. This last method was adopted, the calculations from Dunkerque to Evaux depending on the Melun base, and the southern part from Evaux to Montjouy depending on the Perpignan base.

The arc was calculated by Legendre, Traller, Van Swinden and Delambre, each working independently and pursuing methods a little different in details. The various results obtained by them were submitted to the committee for approval and selection. The mean of the results of the meridian distances as obtained by four different calculators was adopted by the committee. These general results are shown in the following table:

Station	Latitude	Interval			Distance in toise	Value of 1°	Mean latitude		
Dunkerque	51 02 09.20								
		2	11	19.83	124945.18	57076*	49	56	30
Pantheon	48 50 49.37								
		2	40	06.82	152291.48	57066	47	30	46
Evaux	46 10 42.54								
		2	57	48.24	168849.10	56979	44	41	48
Carcassonne	43 12 54.30								
		1	51	09.34	105498.96	56945	42	17	20
Montjouy	41 21 44.96								
total		9	40	24.24	551584.72	57018	40	11	58

* This is given (51. vol III, p. 428) as 57276, an evident mistake.

A comparson of this arc with that of Peru gave the ellipticity as $\frac{1}{334}$, which was used in the establishment of the meridian quadrant length. Using the above given value of the arc, there resulted for the length of a quadrant of the meridian, considering the earth as a spheroid of revolution, 5,130,740.0 semi-modules, the ten-millionth part of which is 0.513,074 semi-modules. Calling the semi-modules equal to 864 lines, it results that the metre equals 443.296 lines, or 3 feet 11.296 lines, in employing the toise of Peru at a temperature of 13 Reaumur.

The results of comparison between an English standard made by Troughton and said to be identical with the standard made for Sir George Schuckburgh, gave the metre of platinum equal to 29.3781 English inches and the metre of iron equal to 39.3795 English inches, the temperature being 12¾° Centigrade. This is given as 12¼° C. in another place. Earlier comparisons between the toise of Peru and the English standard gave the toise equal to 76.7394 English inches at a temperature of 13½° Centigrade.

EXTENSION OF THE FRENCH ARC.

It had been the conception of Delambre and Mechain, as a result of their work in determining the length of the metre, to extend the triangulation southward, if possible, through Spain and to the Balearic Islands. The death of Mechain, however, prevented the carrying out of their plans. The work was undertaken a few years later at the request of the Bureau of Longitudes, Messrs Biot and Arago being appointed to take charge of the work. The sanction and assistance of the Spanish government was obtained, two persons being delegated to assist in the work. No new base lines were measured, but the triangulation was extended to Formentera, and astronomical work done there. This island is at the southwest extremity of the Balearic Islands and about 60 miles from the mainland. The stations for connecting the island to the mainland were selected, the island station being on Ivica, an adjoining island to Formentera, and the work of observation undertaken really as an experiment. For signals, reflecting lamps were used and the work as a consequence was performed at night.

At the island Ivica an attendant was left to see that the lamps which had been oriented by the compass were properly cared for. At the mountain Desierto de las Palmas, on the mainland, a hut had been erected and the instrument set up for work. For over two months, at the end of the year 1806, they occupied this station, vainly watching for the lamp signal at Campivy (island of Ivica). They were unable to determine whether it was inaccuracy of direction of the pointing of the signal lamp or lack of power of the instrument which prevented their seeing the signal. As a final solution they resolved to point their telescope upon the island when visible in the day-time, and see whether at night the signal lamp would appear in view. Accordingly on a clear evening when the islands were visible at sunset, the telescope was pointed on what they considered to be the mountain station and clamped. As darkness came on they discovered in the field of view the signal lamp showing very faintly.

Their difficulty had been one of direction, they not having been able to recognize the signal lamp as they swept the horizon with their telescope. Having once found the signal and noted the angle, they were enabled to make further observations on clear nights. The observations were finished at this station in January, 1807, and by April all the angles of the island connection had been observed.

Arago completed the other work during the year. A very large number of observations on each angle were taken, usually about 100 sets, each set varying anywhere from 2 to 28 repetitions. These were grouped according to the different days' work, and the arithmetical mean taken. Vertical angles, or rather zenith distances to distant stations were also taken, from which the elevations would be determined trigonometrically.

Latitude observations were made at Formentera, during 1807 and 1808, by both men using a new repeating circle of Fortin of Paris. This instrument was provided with a level which was independent of the limb of the instrument. In making the observations, instead of always trying to keep the bubble in the center of the tube, the readings of the bubble extremities were taken and calculations made for the corrections due to the inclination of the axis of the instrument. The value of one division of the bubble tube was found by noting the change in the reading, by turning the telescope through a certain angle, as determined by the reading of the circle. A very large number of zenith distance observations, nearly 4,000, were made on the stars α and β Ursae Minoris.

The latitude, as deduced from Polaris for about 1300 observations at each upper and lower culmination, differed by a little over two seconds, and gave as a mean 38° 39′ 56.00″. Those from β Ursae Minoris at upper and lower culmination, about 700 observations each, differed 0.2 seconds and gave as a mean 38° 39′ 56.02″, or as a mean from both stars 38° 39′ 56.01″, to which was added 0.1 seconds for reduction to the trigonometric station. These results would indicate considerable accuracy, but as a matter of fact they were quite a little in error. All observations had been made with stars on one side of the zenith, and thus no chance allowed for elimination of errors. The latitude was reobserved by Biot, in 1825, and he obtained as a result from using stars both north and south of the zenith, the latitude 38° 39′ 53.17″.

WORK IN RUSSIA AND THE ARCS OF MERIDIAN MEASURED BY STRUVE AND TENNER.

As early as 1737, Joseph Delisle, astronomer of St. Petersburg, influenced by the Royal Academy of France, obtained from the Empress Anne a sanction of his project for measuring an arc which could be made to include some 22 or 23 degrees of latitude. A base was measured, in 1737, on the ice of the Gulf between Doubin, on the island of Ritusari (the present site of Kronstadt), and Peterhof, on the mainland to the south. The measurement of this base, which was about 14.4 kilometres long, was made by means of wooden rods. In 1739, the base line was connected by triangulation with several stations, and after that nothing was ever done. The reasons for discontinuing the work are not definitely known, the results as regards measurement of the base and observed angles were never published, and the project was forgotten. It was not until 1844, that M. O. Struve, in looking over papers preserved at the Observatory of Paris, discovered Delisle's manuscripts.

In 1814, M. de Lindeman, later director of the observatory at Seeberg, proposed the measure of a three degree arc along the shores of the White Sea, as a means of settling the discordance between the results of Maupertuis and those of Svanberg, which had just been published. The project was favorably acted upon by the higher officials, but was abandoned because an agreement could not be reached regarding the instruments to be used. Lindeman insisted that the instruments of Reichenbach of Munich should be used, while the officials insisted that the instruments be made at St. Petersburg.

The real work was started in 1816, by F. G. W. Struve and General de Tenner, each working independently of the other. As early as 1812, Struve had investigated in the province of Livonia the feasibility of a trigonometric survey, and had measured by means of wooden rods a short base, and had observed a few angles. Further investigations were stopped on account of the war. In 1816, he was appointed to take charge of a trigonometric survey which was executed during the years 1816-1819. In the process of the work several auxiliary base lines were measured, latitude and azimuth observations were made, and the work was terminated by the measurement of a principal base on the ice of Lake Werz-Jerw, in 1819. The terminal stations were on the shore. The angles between the signals were observed with a reflecting sextant, of a 10-inch radius, which was made by Troughton of London. The vertical angles were measured with a sector made at Dorpat under the direction of Struve himself.

This work, while not as accurate as that which followed, was very well done, as was shown by the agreement with the more precise work later on. The trigonometric work once being started, Struve interested the officials and obtained permission to undertake the measurement of an arc of about 3° 35′, comprised between Hogland, an island in the Gulf of Finland, on the north and the village of Jacobstadt on the south. This was accomplished during the years 1822-1827, the work being terminated by the careful measurement of a base and its connection to the triangulation system.

Astronomic observations were made at Hogland, Jacobstadt and Dorpat by means of a vertical circle. The results obtained were:

Latitude	Jacobstadt	56° 30′ 4.562
"	Hogland	60° 05′ 9.771″
Amplitude of arc		3° 35′ 05.209″ ±0.121″

The distance found by triangulation was 204 819.554 toises ±1.146t.

The first work of Tenner consisted in conducting a trigonometrical survey in the province of Lithuania, and this was extended, in 1825, to the measurement of a meridian arc of about 4½°. The triangulation depended on two bases in the vicinity of Bristen and Belin. The amplitude was obtained after the manner of Bessel by observations made for latitude in the prime vertical. The work was completed in 1827, the same year that Struve completed the arc to the north. The situation of these two arcs was such that the northern terminal of Tenner's arc at Bristen was in nearly the same latitude as Jacobstadt, the southern terminal of Struve's arc, and about 20 miles to the east of it.

The suggestion was a natural one that, as these two were so near, they should be united, to give one long arc.

This was done in 1828 and gave as a result an arc of 8° 02′ between Hogland and Belin. The terrestrial distance was 459 363.008 toises. These results were published in 1832, and this arc was one of the ten used by Bessel in determining the figure of the earth.

There was now about 5½° separating the Lapland arc and the Russian arc. Struve sought and obtained permission, in 1831, to extend the triangulation northward, to connect with the Lapland arc, with the idea of still further extending it on the north to the Arctic ocean. The field work was started is 1832 and continued up to 1845. Two bases were measured, one in 1844 near Elimä, a little north of the gulf of Finland, the other in 1845 near Uleaborg on the border of the Gulf of Bothnia. The junction with the Lapland arc was effected in 1844, and astronomic observations of azimuth and latitude were made at Tornea.

During this same time the triangulation had been extended from the south end of the arc still further south to the Dnestre River. In 1844, further aid was given to the extension by the imperial order to continue the arc south to the Danube River. This work was placed in charge of General Tenner. It was required, also, that the Central Observatory co-operate in the measurement of the meridian by the determination of latitudes at different points. The work of calculation was placed under the charge of the latter, and all previous work was to be given into its care. This served to place in Struve's hands all the original material of the trigonometric work. A readjustment and calculation of all the work between Tornea on the north and Ssuprunkowzi on the Dnestre River was made, with an equal representation of the bases. This result gave an arc of 17° 05′ 33″, which was used by Bessel in his final determination of the figure of the earth.

The work of the southern extension was completed in 1848-1849. Two bases were measured and two stations were occupied for latitude and azimuth. The southern terminal of the arc was at Staro-Nekrassowka. In 1844 Struve was sent by the Academy of St. Petersburg to Stockholm to confer with the scientific men of Norway and Sweden, with a view of extending the system to the Arctic ocean. His mission was successful, and two commissioners, one from Sweden and one from Norway, were appointed by the King. These commissioners were to make a reconnaissance and report on the feasibility of the plan.

The Swedish commissioner examined the country from Tornea northward to Kautokeino, the Norwegian commissioner southward from Cape North to the same place. They then met in Stockholm and reviewed the situation. They pronounced the project feasible, and gave in general a plan for the triangulation. The work was commenced in 1846 and completed in 1850, the southern part being executed by men from Sweden under the direction of Mr. Selander, while the northern part was executed by men from Norway under the direction of Mr. Hausteen. The northern point selected was Fuglenaes on an island near Hammerfest. Cape North, a point 1-2 a degree further north, was reluctantly given up. On account of the habitual storms and severe weather it was felt that astronomic observations would be difficult to obtain.

The instruments used by the Swedish observer were a universal instrument and a vertical circle of 12 inch radius, both of which were read by microscopes. Latitude observations were made by these observers at their most northern point, Stuor-oive, latitude 68° 49′ N., some 500 observations being made upon the zenith distance of Polaris. Azimuth observations were also made. The astronomic work at Tornea was left incomplete in 1850 owing to the inclemency of the weather.

The Norwegian observer used a small universal instrument of Repsold, with a 6 inch circle, which was divided to 10 minutes and was provided with micrometer microscopes reading to seconds. A larger instrument could not well be used owing to the difficulties of transportation. The scarcity of wood in that northern latitude was a great inconvenience and necessitated the transportation, from a long distance, of the wood used for the signals. Communication with the island stations along the shore was also arduous, but in spite of these facts the angles of the 15 Norwegian stations were measured in the years 1846-1847.

The base line in Nowegian territory and its connection to the triangulation was made in 1850. At the request of the Norweigan commission, the Central Observatory of Russia engaged in the measure of the base line near Altengaard. The base measuring apparatus of Struve with other instruments were sent and the measurement made under the direction of Mr. Lindhagen, who had been delegated by the Central Observatory. The Norwegian commission furnished transportation of instruments and aided in the expense.

The base measured July 5-12 was 1155 toises in length. Astronomic observations were also made at Fuglenaes. The results of the work, both Swedish and Norwegian, were placed at the disposal of Struve for calculation. In the Swedish operations, which were left incomplete in 1850, there still remained the latitude observations to be made and a base line to be measured and connected. These were taken up again in 1851 with the co-operation of the Russian Observatory. By this means the same instruments were employed for astronomic work that had been used at the northern station.

In general the triangulation stations were those previously used by Maupertuis or Svanberg, but a base line was measured in a different place. The two former base lines had been measured upon ice, but this was measured on land between Tornea Elf and Mount Arasaksa. Astronomic observations were made at a station between Neder Tornea and the old trigonometric station of Kokko-maki during June and July. The latitude observations were made with a Repsold vertical circle placed in the prime vertical. Azimuth and time observations were also made. During the observations, the instruments were supported upon solidly constructed masonry. The base line, of which the ends were definitely marked, was measured twice in August and was 1520 toises long.

During the years 1852-1854 several new observations for latitude at the southern station of the arc were made with the same instruments previously employed at the north, for it was felt that this could give greater weight to the work. Also one southern base was remeasured, as there was a question of comparison of the measuring rods with the standard.

A second astronomic expedition to Finland was made in 1852 to determine a new astronomic point at about the middle of the northern part of the arc. As a result of conference between the Central Observatory of Russia and the commission of Sweden and Norway, all the details of the work done by them were placed in the care of the Central Observatory for a complete adjustment and calculation of the arc.

Standard unit of length. The unit standard of length employed as a base of all the calculations was a toise made by Fortin of Paris. Its length was the same as the toise of Peru, as is shown by the following certificate of the Bureau of Longitudes. "Je sousigne, membre de l'Institut et du bureau des longitudes, certifie avoir comparé la Toise en feu construite par Fortin et destinée a Monsieur Struve, à la Toise de Pérou qui est conservée dans les archives de l'observatoire Royal. Les deux toises m'ont paru parfaitment égales; la comparateur dont je me suis servi m'auroit fait connaitre une differénce de la deux centième d'un millimètre. Paris, le 14 Novembre, 1821. Arago." (64. vol. 2, p. 400.)

From this standard known as F, a standard known as N was constructed. This was of iron about 2 toises in length, 1 1-4 inches square in general cross section, terminating in cylindrical ends, which were tempered. The diameters of the cylindrical ends were 2 1-2 lines, and they presented an exterior surface which was a little rounded and perfectly smooth. The distance between the center of the two end curved surfaces was the length of the standard, when the temperature was +13.0°R. or 16.25°C., and the bar supported at its quarter and three quarters points. Calling the length of the standard F equal to 864.00 lines, the length of N was 1728.01249±0.000 71 lines at 13.0°R., a little over two toises.

This standard was the observatory standard, although it was used a few times in the field, and from it two prototypes, known as P and Q, were made to use as general field standards in connection with the base measurements. Their values were

P = 1727.99440 ± 0.00019 lines at 13.0°R.
Q = 1728.01991 ± 0.00077 lines " " R.

The value of another standard scale known as T and used by General Tenner for standardizing his measuring rods was in terms of $N = 945.75779 \pm 0.00038$ lines at 13.0°R. On these three standards in terms of N, and consequently of F, depended all the calculations.

Base apparatus. Struve's base apparatus consisted of four measuring rods known as A, B, C, and D. Each rod was of forged iron, 12 feet long, 15 lines wide, and 15 lines thick. One end was turned down and terminated in a small tempered cylinder, the end surface of which was slightly rounded and smoothly polished. The other end carried a contact lever of steel which was pivoted to the bar. This lever, in the form of a right angle with one side long and the other short, was pivoted at the right angle. The short arm in the direction of the bar terminated in a hemispherical end. The longer arm carried an index mark and passed over a graduated scale. A spring acting against the lever kept it in a stable position.

The length of the rod was from the convex surface of one end, to the hemispherical surface of the contact lever, when the reading of the lever was at scale 15 of the vernier. The lever was encased in a brass box rigidly attached to the iron measuring rod, which allowed the hemispherical knob to project, and which was provided with an opening covered with glass for the reading of the vernier. The rods, which were in a wooden case, were supported at the quarter and three quarters points, the supports being such that if necessary a longitudinal motion of about an inch could be given to the rods by means of a lever on the outside. Two thermometers, the bulbs of which entered into drilled holes in the measuring rods, rose above the top covering, and were in sealed wooden cases with glass fronts for reading.

Near the center of the top, on the outside of the wooden case, were two supports upon which the level was placed in order to determine the inclination. The exterior of the wooden case was well oiled and painted white, as was also the inside. Everything inside, including the rods themselves and the thermometers, was covered with thick cotton, thus tending to prevent changes in the air from affecting the temperature of the bar. The only part exposed to the air was the small cylinder at one end and the contact lever in its brass case at the other.

In measuring, the rods were supported on solidly constructed tripods about 2 1-2 feet high. The boxes were firmly held in position by three vertical wedging screws and were aligned by two horizontal screws. The measurement was by direct contact of the measuring rods. That is, the projecting cylindrical extremity of one rod was brought to press against the hemispherical knob of the contact lever of the other rod until the vernier reading was 15 of the scale.

The sector used for determining the inclination consisted of a level bubble attached to an arm which was pivoted at one end, the other end being raised by a vertical screw, and the reading was obtained from a graduated scale and vernier. For terminating the work of any day, a large iron picket was driven into the ground to a depth of about 2 feet, leaving exposed about 1 foot. The top of this stake carried a projecting arm which was grooved, and in this groove slid a square metal piece having a small silver dot, the center of which was placed directly under the terminating measuring rod.

This was accomplished by setting up a theodolite at right angles to the measured line and bringing the extremity of the measuring rod down (by means of the cross-hairs of the telescope) and sliding the metal piece containing the silver dot into position. The piece was then firmly clamped by screws and the point secured. In commencing work the process was reversed. For comparing the measuring rods with the field standard P or R, devices similar to those that terminated the ends of the measuring rods were used.

The standard P abutted at one end against a cylindrical bar with a rounded end and at the other end against the hemispherical knob of a contact lever. To the first, which could be moved longitudinally, was applied pressure enough to bring the contact lever to the proper reading, at which point the piece was clamped. The standard was then removed and a measuring rod substituted. The difference in the vernier reading of the contact lever then indicated the difference in length between the two. In the place of the contact level, sometimes a micrometer microscope was used, the difference being obtained in revolutions of the head.

Base apparatus of General Tenner. This was modeled after and an almost exact imitation of that made by Borda (see page 232) and used by Delambre and Mechain. Their apparatus was made originally of brass and iron joined at one extremity, and fitted with a vernier at the other. From experiments made by General Tenner, the expansion was found to be irregular; and the brass strip was discarded and a mercurial thermometer substituted. The measuring rod then consisted of a bar of forged iron, 14 English feet in length, 0.85 of an inch wide and 0.3 of an inch thick. This was supported upon a board and provided with a covering protecting it from sun and air, so that nothing was exposed except the two terminal points.

The bar was fixed to the wood at one end but could expand freely longitudinally. At the free end was a sliding scale for bringing it into contact with the adjoining bar, the scale being read by a vernier and microscope to thousandths of an inch. Each rod was provided with a thermometer, the bulb of which was encased in a globe of iron which was placed in contact with the bar. The stem of each thermometer projecting above the covering permitted the reading to be taken easily. All thermometers had been carefully verified. The process of measuring was similar to that used by Delambre and Mechain with this addition, that portable screens covered with white cloth were placed along the line, and the measurements were made in the shade.

Bases measured. Seven bases were measured with the apparatus of Struve and three with that of Tenner.

Base measures with the apparatus of Struve:

Base	Year measured	Length in toises	Length corrected for temperature, inclination, etc.
Simonis	1827	2315.2	2315.17257± .792u
Elima	1844	1348.8	1348.75014± .793u
Uleaborg	1845	1505.3	1505.31781± .799u
Romankautzi	1848	2910.2	2910.22602± .914u
Taschbunar	1852	2770.3	2770.26946± .842u
Alten	1850	1154.7	1154.74458± .793u
Ofver Tornea	1851	1519.9	1519.85006± .729u

Base measures with the apparatus of Tenner:

Ponedeli	1820	6055.4	6055.25206±3.16u
Ossownitza	1827	5720.0	5719.78406±3.04u
Staro-Konstantinow	1838	4564.2	4564.17347±3.04u

In the above table $u = \frac{1}{1000000}$ of the length of the arc measured.

The probable errors have been obtained by considering errors due to (1) alignment, (2) inclination, (3) error of comparison between field standard and observatory standard, (4) error in comparison between measuring rod and field standard, (5) error in reading contact lever, (6) personal errors, (7) errors due to temperature. These ten bases were further reduced to sea level, six being referred to the level of the Baltic Sea, and the remaining four to the level of the sea near which they were located. The final lengths were:

Base	Length in toises	Referred to the level of the
Simonis	2315.12307±0.00187	Baltic Sea
Ponedeli	6055.16232±0.01925	" "
Ossownitza	5719.64316±0.01773	" "
Staro-Konstantinow	4563.97153±0.01438	" "
Romankautzi	2910.09451±0.00309	" "
Tashbunar	2770.24614±0.00274	" "
Elima	1348.74573±0.00109	Gulf Finland
Uleaborg	1505.83864±0.00121	" Bothnia
Ofver-Tornea	1519.83864±0.00113	" "
Alten	1154.74393±0.00093	Arctic Ocean

Signals used. Most of the signals used by Struve were of tripod form, the mast being circular, from 8 to 12 inches in diameter, or square, 4 x 8 inches, the size used varying according to the length of the lines. They were carefully erected, strongly braced, and well plumbed. The heliotrope was used some on very long lines. The signals erected by General Tenner between the Dena and the Danube were pyramidal or of the scaffolding type. The pyramidal signals terminated in a short mast about 2 feet long and 4 inches square. The sides of the top were boarded in for a short distance down and painted black. The scaffold signals, which also served as instrument stations, were square in shape and were braced externally by inclined timbers. The tops terminated in pyramidal roofs similar to the other signals.

For measuring the terrestrial angles, eight different instruments were used during the years 1818-1849. That used by Struve was a universal instrument made by Reichenbach of Munich in 1820. This was used during all the years and last employed in determining the azimuth at the central point, Kilpi-maki in Finland, in 1852. The horizontal circle was 13 inches in diameter and the vertical 11. Both were graduated to 5 minutes and read by verniers to 4 seconds. The large telescope was of broken form, aperture 21 inches, focal length 18, and magnifying power of 60. The cross wires were three in number, two vertical and one horizontal. The smaller telescope for verification was of 16½ lines aperture having two threads at right angles. It was used for horizontal and vertical angles, azimuths, latitude and time.

The other seven instruments were employed in the operations of General Tenner, and consisted of two repeating circles, one made by Baumann and the other by Troughton, a terrestrial repeating theodolite by Reichenbach, an astronomical repeating theodolite by Ertel, and also two repeating theodolites and a small universal instrument. The number and difference in type of the instruments serve to show the progress in geodesy during the years of work. The angles observed in the earlier years were taken with the repeating circle, those of later years with the universal instruments. Thus the earlier angles observed were nearly all made upon the principle of repetition; later they were about equally divided between the method of repetitions and that of positions.

Latitude and azimuth. The complete arc was divided into two parts, one the southern, wholly in Russia, the other the northern in Sweden and Norway. These two large arcs were still further subdivided into partial arcs, the southern part into eight partial arcs and the northern into four. There were thus thirteen astronomical stations for the determination of these twelve partial arcs. For the most part these astronomical stations were identical with the trigonometrical; in some few cases, however, it was deemed advantageous to locate them a short distance away. In the latter cases the results have been reduced to the trigonometrical stations.

The azimuths were obtained by observations on Polaris at elongation, using the universal instrument previously described, the instrument being reversed during the process of a set of observations. In a few instances an instrument of passage was used (probably a transit instrument). The latitudes were obtained both by the use of meridian circles and the universal instruments, these being placed in the prime vertical in some cases; but usually the latitudes were obtained from observing the zenith distances of stars. For the trigonometrical stations in Finland both zenith distances and the prime vertical methods were used.

Computations. The spherical excess was allowed for in all triangles and computations were made after Legendre's method. The errors of measurement of bases, angles, etc., were carefully investigated. Least squares adjustment was made in the triangulation computations. The following table gives a summary of the results obtained. Column (1) gives the name of the astronomical station, (2) that of its resulting latitude, (3) the distance between the parallels of two consecutive stations, (4) the distance from the southern terminal to any other astronomic station.

Astronomic station	Latitude	Distance between parallels in toises	Sum
Staro-	45° 20′ 2.94±.05″		0.000
Nekrassowka		96415.136±0.651	
Wodolui	47° 01′ 24.98±.24″		96415.136±0.651
		98557.988±1.251	
Ssuprun-	48° 45′ 03.04±.10″		194973.124±1.646
kowzi		76751.386±0.710	
Kremenetz	50° 05′ 49.95±.30″		271724.510±2.039
		111219.011±1.008	
Belin	52° 02′ 42.16±.14″		382943.521±2.611
		148809.521±1.426	
Nemesch	54° 39′ 04.16±.07″		531753.042±3.453
		105730.879±0.926	
Jacobstadt	56° 30′ 04.97±.10″		637483.921±3.893
		107280.563±0.675	
Dorpat	58° 22′ 47.56±.05″		744764.484±4.177
		97538.618±0.503	
Hogland	60° 04′ 29.14±.10″		842303.102±4.372
		145713.567±1.072	
Kilpi-maki	62° 38′ 05.25±.08″		988016.669±4.502
		182794.304±1.673	
Tornea	65° 49′ 44.57±.07″		1170810.973±4.957
		163221.904±1.689	
Stuor-oivi	68° 40′ 58.40″		1334032.877±5.539
		113753.906±1.785	
Fuglenaes	70° 40′ 11.23±.06″		1447786.783±6.226

The base terminals and astronomical stations, with the exception of the terminals, were marked permanently by suitable masonry construction. The two terminal points were more elaborately marked. At the southern terminal at Staro-Nekrassowka, the point was marked by a truncated pyramid of cast iron, resting on a cubical masonry foundation, seven feet on a side. On two sides of the pyramid are identical inscriptions, one being in Latin and the other in Russian. The Latin inscription is:

Terminus australis
Arcus meridiani 25° 20′
quem
inde a fluvio Danubio
ad Oceanum Arcticum usque
per
Rossiam, Sueciam et Norvegiam
jussu et auspiciis
Imperatorum Augustissimorum
ALEXANDRI I
et
NICOLAI I
atque
Regis Augustissimi
OSCARIS I
annis MDCCCXVI ad MDCCCLII
continuo labore
emensi sunt
Trium gentium geometrae.
Latitudo: 45° 20′ 2.8″

A corresponding monument was erected by the Norwegian government at Fuglenaes. This was a granite column resting on a pedestal, also of granite. The top of the column was ornamented in bronze and carried a globe of copper representing the earth. Two inscriptions were placed upon the column, one in Latin and the other in Norwegian. The Latin inscription was the same as that on the southern terminal except the substitution of septentrionalis for australis, and the change in the latitude.

THE ARC OF BECCARIA REMEASURED.

In 1809 Baron von Zach investigated certain portions of Beccaria's work. He measured a base near Turin which was 642.49 metres long when reduced to 0°C., the unit of his measurement being an iron metre made by Megele. The computed distances resulting from his work did not closely check Beccaria's, and he was led to believe that the work of the latter was of poor order. He found evident errors in the value of the different assigned latitudes, and from calculations based on the then known size of the earth, he believed the latitude of Beccaria's northern station, Andrate, to be 12″ small, and that of his southern terminal, Mondovi, 17″ large.

Also in 1809, from the extension of the French triangulation through Lombardy, another attempt was made with but partial success, for not all of Beccaria's triangulation stations were recovered. The base at Rivoli was checked upon, however, and the computed length from the Lombardy extension triangulation was found to be 3.76 metres longer than the actual measure by Beccaria.

During the years 1821-1823 a commission composed of Piedmontese and Austrian officers and astronomers carried on a trigonometrical survey in the districts of Piedmont and Savoy. Their work was really an extension of the triangulation system in France, for they started from a triangular side common to that system, and all their subsequent work and computations depended upon the bases measured in France. Their work was primarily for map construction, but in addition to that it afforded excellent opportunities for the measurement of an arc of parallel and also a chance to check Beccaria's meridian arc. No new bases were measured except a short one near Turin for verification purposes.

The longitude of the different stations was determined by observing the local time of gun powder flashes at a distant station. These signals were set off at night at intervals of ten minutes, and were taken at the distant stations by three observers. In this manner they covered an arc of a parallel of 24′ 6.78″. These longitudes, deduced from observation, differed from the computed positions based on the meridian of Paris by errors of 0.39 to 2.08 seconds of time.

Triangulation. The work was carefully laid out, permanent signals erected of square pyramidal form, usually made of stone, but at times of wood. Since these could not be occupied, the eccentric station was also selected and marked, measurements being taken so that the angles could be reduced to the center of the stations. The triangles, 16 in number, were exceedingly well shaped, the smallest angle being 37° and the largest 98°, with minimum and maximum sides of 16030.25 and 48204.82 metres.

The angles of all the triangles were taken independently by the Piedmont and Austrian observers. The instruments used by the Piedmont officers were an 8″ repeating theodolite by Reichenbach and a like instrument by Gambey. The Austrian observers also used an 8″ repeating theodolite by Reichenbach and a 12″ repeating theodolite constructed at the Polytechnic Institute of Vienna. This latter instrument never gave good satisfaction, and nearly all the work was done with the former instrument.

Many repetitions of the angles were taken by both the French and the Austrians. These, while agreeing with themselves, did not always agree well with each other. In one instance they even differed 9.2″, but in most cases the variation was about 1″. This great difference between the two sets of observations was strongly emphasized in earlier days by those opposed to the method of repetitions. For final values of the angles the weighted means of the two observations were taken, and these gave good results. The discrepancy between the sum of the three angles, and 180° plus the spherical excess, exceeded 1″ in only three of the triangles. The computed triangulation checked upon the verification base, near Tésin, within one metre.

The astronomical stations at which observations were made for latitude and azimuth were Mt. Colombier, Mt. Cenis, and Turin. In determining these latitudes a 12″ repeating circle by Reichenbach was used at Mt. Colombier, an 18″ repeating circle by Troughton at Mt. Cenis, and also an 18″ circle by Reichenbach, while at Turin a meridian circle of 3 feet in diameter was used. The latitude of Mt. Colombier was obtained as N. 45°52′ 49.8″, the results of numerous observations on the sun and two stars. At Mt. Cenis the latitude, in 1821, was obtained by the use of the Troughton instrument as 45° 14′ 8.41″
In 1822 by the use of the
Reichenbach instrument as 45° 14′ 7.40″
Giving as a mean value 45° 14′ 7.90″
These observations were made upon the sun and three stars, but the stars were not the same in both years. At Turin, the latitude was observed at three different periods during the year 1821, using the 18 inch instrument of Troughton, and a mean was obtained of 45° 04′ 0.2″. The observations were made upon the sun and four stars, one of which was Polaris. The latitude obtained in 1824-1825 by the use of the 3 foot Reichenbach meridian circle was 45° 04′ 8.15″
as determined from observations on Polaris at upper and lower culmination. The astronomical azimuths, as determined, did not in all cases closely check the computed geodetic azimuths.

At Mt. Colombier they differed 23.7″
At Mt. Cenis they differed 49.7″
At Mt. Turin 4″

The results of this work enabled them to establish between Usson and Milan an arc of a parallel 78,815.7 metres in length. The use of this and the meridian arc from

Formentera to Greenwich gave the ellipticity of the earth as 1/241.1, which is in error about 20%, being too ~~small~~. larger
In checking Beccaria's arc measure they were more successful than those who had made previous attempts. The terminal latitude stations of Beccaria at Mondovi and Andrate were undoutedly recovered. In this they were aided by the description given by Beccaria in his publication. At each place, however, they used different stations, as the old ones did not seem to them to be suitably located. The latitudes at these points were determined in 1821 by means of the 18″ repeating circle of Troughton. Observations were made on the sun, two stars south of the zenith and Polaris north of the zenith. They obtained the following results:

Latitude of Mondovi	44° 23′ 45.38″
correction	−1.01″
Latitude of Beccaria's sector station	44° 23′ 44.37″
Latitude of Andrate	45° 31′ 11.68″
correction	+3.76″
Latitude of Beccaria's sector station	45° 31′ 15.44″

	Remeasured	Revised by Von Zach	Beccaria
Lat. of Andrate	45° 31′ 15.44″	45° 31′ 22.3″	45° 31′ 18.3″
Lat. of Mondovi	44° 23′ 44.37″	44° 23′ 38.0″	44° 23′ 33.6″
Amplitude	1° 07′ 31.07″	1° 07′ 44.3″	1° 07′ 34.7″

There is then a difference in amplitude between Beccaria's results and those of the remeasurement of 3.63″. On comparing the distances between the new stations we have

Length given by Beccaria,	126,356.41 metres.
Length given by remeasurement,	126,394.60 metres.
Difference,	38.19 metres.

We may conclude from this then, that Beccaria's arc was fairly well measured for that time. If the positions of Andrate and Mondovi be computed, starting from the French system, using *a* (the semi-major axis of the earth) equal to 6,376,986 metres and considering the earth an ellipsoid of revolution, there results for the amplitude $1° 8' 14.82''$, which is greater than the astronomic amplitude by $47.84''$, a very large error. It is, strictly speaking however, not all error of observation. The greater part of the difference is due to the large deflection of the plumb-line from the vertical by the Alps mountains. These more modern results, like Beccaria's, have never been considered of sufficient accuracy to warrant their use because of the incorrect amplitudes due to local attraction.

The verification base, previously alluded to, was measured primarily for checking the work of Von Zach. The length at sea level was 641.0990 metres at 0°C. The line was measured with wooden rods. These were 4 metres long, constructed in rectangular form with a hollow cross section, being made of 4 boards joined together. On the vertical sides at one end was a dot, while at the other was an open brass frame carrying a vertical thread. In measuring, the rods were supported, the measuring dot at one end of the bar being brought into coincidence with the vertical thread of the adjacent bar.

The result of this work was the preference it gave to Beccaria's work over that of Von Zach's; for when a common triangulation line was computed from the four bases, namely, the French base, Beccaria's base, the remeasured base, and Von Zach's base, it was found that the computation from Von Zach's base did not agree well with the others.

THE ARCS IN INDIA.

Bengal arc. The starting of trigonometrical work in England, in 1787, by General Roy, led the East India Company of London to make an arc measurement in Bengal. This work was done in the years 1790 and 1791 by Reuben Burrow, of their corps of engineers. The work had been nearly completed when Burrow died in 1792. The rather incomplete results were published in 1796 by Isaac Dalby.

The instrumental equipment consisted of a theodolite, an astronomical quadrant by Ramsden, a Hadley sextant, several chronometers and a 50 foot steel chain by Ramsden. Burrow desired and tried to obtain a zenith sector from London, but failed.

The measurement of the arc of a parallel and also of the meridian was made in the vicinity of Cawksally. As the work was not intended for map construction, nothing was done except the mere measurement of the line, which was measured its full length by the use of the tape or bamboo rods. The arc of the parallel was measured eastward and westward from Cawksally. The distance measured to the east was about 33½ miles, but the exact figures are not given. The total distance measured westward was 212,670 feet, and of this distance the first 82,678 feet were measured twice, checking within 12 feet. Their alignment was obtained from observations on Polaris.

The difference in time between the two extremities was found by the chronometer to be 2′ 32″, giving the length of 1 degree of an arc of a parallel in latitude 23° 28′ as 335,937 feet, or 55,989 fathoms. The arc measurement was made in 1791, between Poal and Abadanga. The greater part of the distance was measured with bamboo rods 194 feet long.

The result of the measurement was 411,004 feet when reduced to the temperature 62° Fah. The latitude of the stations was determined by the Ramsden quadrant, from observations on stars north and south of the zenith.

59 observations at Poal	gave a latitude of	22 44′ 12.7″
131 " " Abadanga	" " "	23° 52′ 11.7″
with a resulting amplitude of		1° 7′ 59″

Hence 1 degree equals 60,457 feet at a latitude of 23° 18′.

These results have never received any serious considerations, as manifestly the work was not up to the standard of even that time.

LAMBTON'S AND EVEREST'S INDIAN ARCS.

The real work in India was begun in 1800, by the then Major Lambton, who proposed a plan for the survey of a portion of India and received permission from the authorities of the Madras government to undertake the work.

His intrumental equipment at first consisted simply of a 100 foot chain of steel made by Ramsden and a 5 foot zenith sector by the same maker. This steel chain had a peculiar history. It was made exactly like the one used on the Hounslow Heath base in England (see page 56), and was originally sent with Lord Macartney's embassy as a present to the Emperor of China. Being refused by him, it was given to the astronomer, Dr. Dinwiddie, who brought it to Calcutta and offered it for sale with a Ramsden zenith sector. They were bought by Lord Clive, then Governor of Madras, and became the first instrumental equipment of Major Lambton.

This outfit was added to in 1802, by receiving from England a 3 foot theodolite by Cary, an 18 inch repeating theodolite by the same maker, two steel chains by Ramsden, a 3 foot brass scale by Cary and several minor instruments.

The original chain was used for measuring up to 1802 without any means of comparison with a standard. After the receipt of the new chains, one of these served for a time as a field standard, the measurement being made with the first chain. Later the 3 foot brass scale was used as a field standard, for it was found that the chain standard, owing to wear by keeping it clean from rust, was changing its length.

The Standards of lengths governing the measured base lines since the year 1802 were as follows:

1. Cary's 3 foot brass scale. This was 3.15 feet long, 1.51 inches wide and .14 inches thick. It was made by Cary from a scale owned by Alexander Aubert and used to determine the length of the Ramsden steel chains used in measuring bases from 1802 to 1825. No direct comparisons were ever made between it and the subsequent standards, although indirect relations were obtained. Its value relative to the other scales in England was known, however, provided it was considered an exact copy of the Aubert scale. This assumption was made by Capt. Kater in 1821, when he made his comparison of lengths. The validity of this assumption may well be questioned, as it is practically impossible to make two scales absolutely equal.

2. The 10 foot iron standard bars A and B and the 6 inch brass scales A and B. These standards were constructed in England under Colonel Everest's direction and were probably brought to India in 1832. The iron standards were 122 inches long, 0.9 inches broad and 2 inches thick. Each standard was encased in a box, and was supported at the one quarter and three quarters points. The extremities of the bars were cut away to half their depth so that the surface of the platinum pins, marking the 10 foot distance, came in the neutral axis of the bar. In

the upper surface, at the one quarter and three quarters points were drilled holes for the insertion of thermometers.

The brass scales A and B were each 10.35 inches long, 2 inches broad and 0.5 inches thick. Marks were engraved on silver pins set in the brass 6 inches apart. Each was provided with a thermometer and a micrometer for comparing the 6 inch distance on the scale with the distance between the visual axes of the compound microscopes of the base apparatus. Several comparisons between the respective standards were made in India by Colonel Everest. In 1831 iron standard B was sent to England and compared with the Ordnance Survey standard O_2. In 1843, both the iron and brass standards B were sent to England and deposited with the Ordnance Survey. The iron and brass standards A remained permanently in India. This second set of standards governed the measure of eight bases between the years 1832 to 1863.

3. The 10 foot steel standard the 1_8, 10 foot bronze standard 1B, and the standard steel foot 1F. These were made in 1864, by Troughton and Simms. The two 10 foot standards were similar in dimensions and sections, the latter being girder shaped. They were 122.9 inches long, of total depth 2.55 inches, breadth of flanges 1.57 inch, breadth between flanges .74 inch, depth between flanges 1.55 inch. The upper surface of each bar was divided into six spaces by seven gold pins, which were inserted in the bar. These pins were set, one at the center, and at points one foot, two feet and five feet, on each side of the center. The exact distance was determined by lines drawn on the gold pins transverse to the length of the bar. Each bar rested upon eight rollers, and in its top surface provision was made for the insertion of three pairs of thermometers.

The standard steel foot was 13 inches long and one inch square in cross section. Lines marking the inch divisions and $\frac{1}{20}$ inch subdivisions, at each extremity, were engraved on inserted gold pins. Careful experiments were made to determine the expansion of these different standards and their relations to each other.

Period of 1800-1830. The work naturally divides itself into two periods, that prior to and that following 1830. The period up to 1830 may be considered as that of beginnings.

Base measurements. During this period the Ramsden chain was used to measure 12 base lines. Part of the measures were made with the chain supported in deal coffers with a given tension applied, the details of arrangement and work being precisely similar to that used on the Romney Marsh and Hounslow Heath bases in England already described. At other times the ground was leveled, the chain placed directly upon the ground and stretched by a pair of small capstans, but no provision was made for a constant tension as was the case when the tape was supported in coffers.

The Bangalore base, measured in 1800, seems never to have been connected with the triangulation, and a better located one was obtained in 1804. The lengths and details of these different bases were published in the Asiatick Researches, Vol. VIII to XIII, and in Everest's work, "An account of the measurement of an arc of the meridian between the parallels of 18° 3′ and 24° 7′, London, 1830," but neither of these sources has been available to the author.

Instruments. The instruments used for the triangulation were the two previously referred to, namely, the 36-inch and 18-inch theodolites of Cary. This 36-inch instrument was modeled after the great theodolite of Ramsden. The horizontal circle was of 36 inches diameter, the vertical circle 18 inches and both were read by means of two microscopes.

The telescope had a focal length of 37 inches, an aperture of 2½ inches with a magnifying power of about 66. In 1808, this instrument was badly injured. While being lifted to the top of a pagoda at Tanjore, the rope which was keeping the instrument away from the wall broke, precipitating the instrument against the wall, breaking the case and bending the limb to such an extent that it was at first thought to be past repair. Colonel Lambton took the instrument to Bangalore and with the help of the native artificers of the ordinance establishment, succeeded in six weeks' time in restoring the instrument to fairly good working order. To just what extent the graduations were injured never seems to have been determined.

The 18-inch theodolite had both its horizontal and vertical circles of 18 inches diameter, the former read by three and the latter by two microscopes. The focal length of the telescope was 18½ inches with a magnifying power of 40.

The signals used were, for the most part, opaque, being erected masts, flag staves, or cairns of stones. Occasionally blue lights were observed at night, but most of the work was done during the daytime. About 1822, Captain Everest experimented with night signals. These were crude affairs and consisted merely of an earthen cup, about 6 inches in diameter, filled with cotton seeds steeped in oil and rosin. They were burned under a sheltering, inverted, earthen vessel of about 2½ feet in diameter, having an aperture toward the side of the observer. These gave good results and led later to the introduction of night signals as a customary method. The angles were taken with the instrument sheltered and by the direction method. Indeed this might be considered in India as a result of the injury to the great theodolite, for after its accident Captain Everest adopted this method in order to eliminate the unknown errors of his circle.

The astronomical work was done with a Ramsden zenith sector, which was practically identical with the one used in England except that it was smaller in size. The telescope was about five feet long, and the arc was 20 degrees in extent, being graduated every five minutes. Smaller readings were obtained from the micrometer. No levels were provided, and verticality was obtained by the use of the plumb-line. The amplitude of the first arc was determined by taking the difference of the zenith distances of the star Aldebaran as observed at both places. In order to eliminate errors, the observing was done with the plane of the sector toward the east one night and toward the west the following night.

In the earlier part of the work, the azimuths of the triangulation lines were determined from observations at elongation on Polaris alone. Later, two or three polar stars were used at elongation. The results of Major Lambton's astronomical and triangulation work may be briefly stated as follows. As a result of the work of the years 1802-1805, the length of a degree perpendicular to the meridian was found to be

in latitude	12° 32′ 12″	61,061.0 fathoms
" "	12° 55′ 10″	60,743.8 "
" "	12° 55′ 10″	60,751.8 "

The first meridian arc was between Trivandeporum and Pandree. The amplitude from the difference of zenith distances was 1° 34′ 56.428″, and the computed triangulation distance was 95,721.32 fathoms, giving 1 degree equal to 60,494 fathoms for a middle latitude of 12° 32′. In 1805-1806, an arc of about 2° was determined between Dodagoontah and Patchapolliam which made 1 degree equal to 60,530 fathoms for a middle latitude of 1° 55′ 59″. ? This arc was later extended northward to P̄avagada, making 1 degree equal to 60,466 fathoms for a middle latitude of 12° 33′ 9″.

It will be noted that the values of the degree decreased as the latitude increased. This was contrary to principle, and the results troubled Lambton, He believed them due, however, not to the inaccuracies of his work, but to local attraction, so consequently, instead of revising his work, he proceeded to select new stations which he thought might be free from local attraction.

In 1809, the arc was extended southward, giving an amplitude of about 2° 50′ between Patchapolliam and Punnae, with a resulting value of 1 degree as 60,472.83 fathoms in a mean latitude 9° 34′ 44″. In 1811, the work was extended northward to Namthabad in latitude 15° 6′, making an arc from Patchapolliam to Namthabad of about 4° 6′, giving 1 degree equal to 60,487.56 fathoms for a mean latitude of 13° 02′ 55″. Lambton considered that the astromical observations at Punnae, Patchpolliam and Namthabad were less liable to be influenced than the other stations; accordingly he rejected the other observations. As a result of dividing the whole arc between Punnae and Namthabad by its amplitude, he arrived at the value of the mean degree at latitude 11° 37′ 49″ as 60,480.50 fathoms. Using this in comparison with the English, French, and Swedish arcs, he found the mean value of the ratio of the polar to the equatorial radius as 301 to 302, whence he obtained the length of a degree on the equator as 60,857.9 fathoms. During the following years, the triangulation was extended northward, and in 1815 they had arrived at Damargida.

In 1818, the survey work was transferred from the Madras government to the general government of India. Lieutenant Colonel Lambton still remained at the head with Captain George Everest as his principal assistant. The work was carried on until the death of Lambton in 1823, when Everest succeeded him as superintendent. Owing to the ill health of Everest no work was done after 1825, and he returned to England; and while there published an account of Lambton's and his own work under the title, "An account of the measurement of an arc of the meridian between the parallels of 18° 3′ and 24° 7′. London, 1830." His determination as to the form of the earth will be given later under the treatment of the earth as a spheroid.

Period 1830-1850. With the return of Captain Everest, in 1830, to take up the work again, a new era commenced in the survey. While in England new instruments were made, including a Colby base-measuring apparatus, which has been used on bases measured since that time. This instrument was constructed similar to the one previously used for the bases measured in England. No further description of them will be given save that there were six such compound bars, and all measurements were made horizontally under the shelter of tents, the component boxes being supported on trestles during the measurement. Other general details were those of the work on the English arc. The following table gives the base lines measured with this apparatus.

Indian bases measured with the Colby Base Apparatus.

Year of measurement	Name	Geog. position Lat.	Long.	Length at sea level in terms of iron standard A
1831–1832	Calcutta	22° 40′	88° 25′	33959.9174 ft.
1834–1835	Dehra Doon	30° 18′	77° 58′	39184.0321 "
1837–1838	Sironj	24° 7′	77° 51′	38413.5311 "
1841	Bider	17° 56′	77° 37′	41578.5475 "
1847–1848	Sonakhoda	26° 17′	88° 17′	36685.7946 "
1853–1854	Chuch or Attok	33° 55′	72° 29′	41345.4219 "
1854–1855	Karachi	24° 56′	67° 13′	38624.3215 "
1862–1863	Vizagapatam	17° 58′	83° 15′	34778.3908 "
1868	Bangalore	13° 3′	77° 40′	36083.6258 "
1869	Cape Comorin	8° 15′	77° 45′	8912.5279 "

Reduction to sea level was made for all bases, the altitudes being obtained for the earlier bases from elevations determined trigonometrically; for the later bases by direct leveling. The terminals of all bases were marked by appropriate masonry structure, usually containing a brass plate with a circle and concentric dot to show the exact terminal mark. The base Dehra Doon was measured twice, that at Cape Comorin four times, the others only once.

The new instruments brought out by Captain Everest consisted of a 3 foot theodolite by Troughton and Simms, and two astronomical circles of 3 feet radius. This 3 foot theodolite was, in general detail, like those previously described. It had an azimuth circle 34 inches in diameter, graduated to 5 minutes, and was read by five micrometer microscopes. The vertical circle was 18 inches in diameter and the focal length of the telescope 39.4 inches. This instrument never gave good satisfaction until it was sent to their own workshop and repairs made on several original imperfections.

The original Cary instrument that had been damaged and partially restored by Colonel Lambton, was now fully remodeled, a new complete azimuth circle was put on, the two original microscopes increased to five, and practically a new instrument made of it; and it was considered the equal of the new Troughton and Simms instrument. After its reconstruction it was known as Barrow's 3 foot theodolite, from the name of the man who rebuilt it. With these two theodolites most of the triangulation of the following twenty years was done.

Signals. The success with night signals in earlier years induced Captain Everest to bring back from England a number of Argand lamps with parabolic reflectors, and also a number of heliotropes, thus enabling them to take advantage of all kinds of weather. At first the Argand lamps did not give good satisfaction, but it was due to lack of proper arrangement in sheltering. When this was remedied they gave good results. Also blue lights were burned at certain intervals, in specially constructed iron vessels, on those lines where a more powerful light was needed. Many built-up signals either of wood, or brick and masonry, were used in the flat country for the main triangulation points. These were on the average about 50 feet high.

The main triangulation of the southern section of the arc from Kalianpúr to Damargida included 37 primary meridional triangles, and gave a computed distance of 2,202,926.196 feet, as determined from the mean of the computations of the original and computation bases. The section of arc from Kalianpúr to Kaliana included 39 meridional triangles, and gave as a distance between the terminals, 1,961,157.117 feet, determined from the two bases governing the work.

Azimuths were determined at the astronomical terminal stations, and also at five intermediate points on the arc Kaliana to Kalianpúr, and at six intermediate points of the arc Kalianpúr to Damargida. More care was taken in determining the azimuths than in former years. For this work a set of circumpolar stars was made out and these were observed before and after elongation, the time being noted in order that they might be reduced to elongation. The same number of observations were made on each side of the elongation, and the instrument reversed in the process of the work.

Astronomical work. The two astronomical circles built by Troughton and Simms proved upon trial, in 1836, to lack stability, and were found to be faulty in certain parts of construction. They were remodeled under Colonel Everest's direction, and provided with more stable supports. As reconstructed, they consisted of a transit axis supporting a telescope, firmly fixed between two vertical circles. These vertical circles were 3 feet in diameter, graduated to 5′ divisions, and were read by four micrometer microscopes, the heads of two being graduated in reverse order from the other two. Each face of the vertical circle was divided into four quadrants, with the graduations of each quadrant running in the same direction up to 90°. The transit axis was supported by cones of brass, and later these were replaced by sandstone pillars with metal bearing caps. These pillars in turn rested upon a heavy azimuthal circle. The instrument as finished was practically a solidly built alt-azimuth instrument.

At the astronomical stations Kalianpúr, Kaliana, and Damargida observatories were built suitable for the reception of the instruments and the needs of the party. The observations were made practically simultaneously at

Kaliana and Kalianpúr, on nights between December 4, 1834, and February 4, 1840. A selection was made of 36 stars, and a rigorous method was adopted for observation, tending to decrease errors; such as reversing the instrument between each star, shifting the microscope to read different parts of the limb, etc. Of these 36 stars, half culminated north and half south of the zenith, and at a maximum zenith distance of 5°.

Simultaneous astronomical observations were made at Kalianpúr and Damargida between November 24, 1840 and January 11, 1842, and here 32 stars were used. The amplitude of the section Kaliana to Kalianpúr was:

From the north stars	5° 23′ 37.74″
From the south stars	5° 23′ 36.36″
mean	5° 23′ 37.05″
The amplitude from Kalianpúr to Damargida was	
From the north stars	6° 03′ 57.27″
From the south stars	6° 03′ 54.67″
mean	6° 03′ 55.97″

The amplitude of Kalianpúr to Damargida differed only 0.197″, being that much larger than what Colonel Everest gave in his report of 1830. This was a remarkable instance of the balancing of errors. Colonel Everest retired from the work in 1843, and published, in 1847, the results of the arc measurements since 1830.

THE HANNOVERIAN ARC.

Concerning this arc, which was the result of connecting the observatories of Göttingen and Altoona, we have on the whole but little information. The astronomical work has been quite fully given by Gauss, but, as far as the author has been able to ascertain, no published report of the measured base line and the resulting triangulation has been made. What we do possess is a description of the base apparatus used in measuring the base near Braack, in 1820.

The base apparatus consisted of three hammered iron bars, each 2 toises in length, $1\frac{1}{2}$ inches square in cross section, terminating in steel ends, one being flat and the other spherical. The bars, counterpoised to prevent flexure, were supported in wooden boxes and so encased as to show only their ends. During the measurement the boxes were supported on trestles, and at the same level if possible. The bars were not brought into direct contact, but a small distance was left between the flat end of one bar and the hemispherical end of the adjacent bar. This distance was measured by means of a glass wedge, the length of which was 48 times the thickness of its back. When the rods were placed at different heights a longer distance was left between the measuring rods, and in this space a metal rod of known dimensions was placed vertical by means of a level. The distances between the terminals of the two bars to this metal cylinder were then taken by means of the glass wedge.

The astronomical work. The instrument used for obtaining the difference in latitude between the two observatories was the Ramsden zenith sector, used by General Mudge on the English survey (see page 70). At Göttingen, it was mounted in the observatory 1.060 toises north and 7.595 toises east of the center of the meridian circle. It was brought into the meridian by the use of the meridian marks connected with the observatory. At Altoona, it was placed in its observing tent outside the observatory, at a point 13.511 toises south and 2.578 toises west of the center of the meridian circle. Here it was brought into the meridian by time observations on the stars at their culmination.

During April and May, 1827, observations on 42 stars were made at Göttingen, and in June the observations were repeated at Altoona. In general, six observations in each position of the sector were made on different nights on each star; the work of one night being with the face of the sector toward the east, that of the following night toward the west, and so on. The amplitude of arc between the two positions of the zenith sector was 2° 00′ 56.52″, and that between the centers of the meridian circles in the respective observatories, 2° 00′ 57.42″.

The latitude of Göttingen was determined principally from observations made, in 1824, upon northern stars at upper and lower culmination, and this taken in connection with the amplitude gave the following latitudes:

Göttingen observatory	51° 31′ 47.85″
Altoona "	53° 32′ 45.27″

From the results of the triangulation, Gauss (67, p. 71) says that Altoona lies 115 163.725 toises north and 7.211 toises east of the observatory of Göttingen. This result depends upon the triangulation line Hamburg-Hohenhorn, of length 13 841.815 toises, which in turn depends on the base measured in Holstein in 1820.

This amplitude and distance gave the value of one degree, at middle latitude, equal to 52° 32′ 16.56″, or 57 127.2 toises. The measuring bars were not directly compared with the French standard, but the errors according to the statement of Gauss would be very small.

THE DANISH ARC.

Information about this arc is also meagre, but it was probably measured about the same time as the Hannoverian, and perhaps a little earlier. Bessel (81, p. 357) gives the amplitude as 1° 31′ 53.306″, and the meridian distance as 87 436.538 toises. Other details are lacking.

THE PRUSSIAN ARC.

This work, done under the direction of Bessel, was performed during the years 1830–1835, and afforded a connection with the triangulation systems of France and England, and also the Russian work of Struve and Tenner.

Base apparatus. These were constructed upon the metallic thermometer principle, and were four in number. Each consisted first of an iron bar made of commercial iron, 2 toises long, 12 lines wide, and 3 lines thick. Upon this iron rod was placed a strip of zinc of equal width but half the thickness of and a little shorter than the iron bar. The zinc strip was rigidly attached at one end, by means of screws, to the iron bar, but the other end was free to expand. Both metals were highly polished in order to reduce friction. Each end of the zinc bar was tipped with steel drawn out wedge-shaped and terminating in a horizontal knife edge. At the fixed end this knife edge projected a short distance over the steel bar. The free end, the iron bar being longer, had fixed upon it a small steel piece terminating in two vertical knife edges, the inner one being a short distance away from the horizontal knife edge of the zinc bar, and this distance varying with the temperature, formed the thermometer. The outer knife edge projected over the end of the iron bar and could be brought into contact with the adjacent bar.

For measuring this varying distance between the two knife edges, a wedge-shaped glass scale was used. This scale was 41 lines long, 0.8 lines thick at one edge and 2.0 lines at the other. On its surface was engraved 120 division lines at ⅓ of a line apart.

The measuring rods were carefully compared with a standard toise made by Fortin of Paris, which had been compared with the toise of Peru by Arago and Zahrtman. Its length was 863.9992 lines. For determining the length of the bars, two knife edges were fixed at a known distance apart, which distance was a little longer than the length of the bar. The bar was placed between them and the distance measured by means of additional short cylinders and the glass scale.

The bars were supported upon seven rollers and encased in a rectangular frame, the ends alone projecting. Provision was made by means of a screw motion for moving the rods longitudinally on their rollers to bring them into contact. For determining the inclination, a level bubble was mounted on the top of the enclosing wooden case. This bubble was hinged at one end, the other being controlled by a screw fitted with a graduated circle, thus acting as a micrometer by means of which inclinations up to 3 degrees could be read.

Thermometers were not considered a necessity, for the temperature of the bars was supposed to be given exactly by the space interval between the two knife edges. Each bar was, however, provided with one thermometer, it being wholly encased in the box, a glass covering in the top of the case permitting its being read.

Base measurement. The location of the base was between points in the vicinity of Trenk and Mednicken. This was a much shorter base than had generally been used, but Bessel considered that a short line extremely carefully measured would bring about as good results as a longer base line.

The measurement commenced August 11th, 1834, the measuring boxes being supported their entire length on trestle frames. The first day 226 toises were measured and the point marked by placing a stake in the ground, and in the top of this stake a needle, which was located by plumbing down from the knife edge of the measuring bar. This point was called A. On the two following days the measurement was completed. The distance Trenk to A was measured twice, that from A to Mednicken twice, and always in the same direction. The first measurement, when the point A was established, was, however, not used in determining the length.

The results of the two measurements when corrected for length of standard, temperature, and level are:

First measurement	934 toises	861.054 lines
Second "	934 "	863.156 "

a difference of about $\frac{1}{8}$ of an inch. The average of these, reduced to sea level, gave the length as 934.993, 124 toises.

The terminal points were marked by underground masonry structures, the precise point being defined by lines intersecting at right angles on the top of a brass bolt, which was firmly imbedded in a granite block. For a surface mark, the masonry was brought up in a hollow form and surmounted with a sandstone cap carrying a bolt similar to the underground mark and vertical above it, thus enabling the point to be easily occupied with the instrument.

Observing instruments and signals. The greater part of the horizontal angles were observed with a theodolite having a 15″ circle. This was an instrument made by Ertel after the design of Scnumacher, and could be used as a repeating instrument if desired.

The telescope was 19 inches long with an aperture of $1\frac{3}{4}$ inches, and was supported by vertical standards attached to the alidade plate and 10 inches high. A striding level permitted the placing of the axis of the telescope horizontal. To the horizontal axis of the telescope was attached a vertical circle of $7\frac{1}{2}$ inches in diameter, having four verniers, and reading to 4 seconds. To this circle was attached a level for obtaining the index correction.

The 12-inch theodolite, made by Pistor and Schick, was of the repeating type, graduated to 5 seconds, and was provided with four verniers. It had no vertical circle.

The signals were in the main of three kinds: 1st, heliotropes, used on the longest lines; 2nd, stationary signals, which were used on the shorter lines. These were of two types, the first consisting of copper hemispheres, the axes of which were vertically over the station. There were seven of these hemispheres, four being of 4 inches diameter and three of 8 inches diameter. These latter were easily distinguishable with the 15-inch instrument up to distances of six miles, being highly polished, reflected the sun's rays and afforded a shining mark on which to sight. There is one serious objection to such signals and that is that all observed angles must be corrected for what is called "phase," since the line of reflected rays will not coincide with the line to the center of the hemisphere. This correction is not constant, but depends on the position of the sun with respect to the observer.

The other class was a form of a tripod signal having a vertical axis, which carried a board 2 feet square, painted white with a vertical black stripe 10 inches wide in its center. This board, by rotation about its vertical axis, could be placed in any desired position. At a few places it was necessary to erect small scaffold signals and observing stands.

In observing the angles the instrument was placed over the point occupied by the heliotrope or reflecting signal. The direction method of observing angles was used. The verniers being placed at zero, a pointing was made at the first object, the alidade was then unclamped, and the telescope pointed at the second object and the four verniers read, and so on around the horizon to the point of starting. The telescope was then reversed and the pointings made in a reverse direction. This constituted a set.

The vernier was then set at 15 degrees and a pointing taken upon the first object, and then in like manner upon the others, as in the former set. The work was continued thus until enough sets had been taken. Where the repetition method was used, the angles were usually observed in sets of 5 and 10, several sets being taken.

Astronomical observations, for azimuth and latitude, were made at three places, Trunz, Memel, and Königsberg. The latitude instrument was a meridian instrument made by Repsold of Hamburg, having a $1\frac{3}{4}$ inch objective, a focal length of 21 inches, a recticule of 5 threads, graduated to 2 seconds of arc and fitted with excellent levels. The previously described universal instrument was used for azimuth observations and also the meridian instrument.

At Trunz, latitude observations were made on

10	different nights	stars north and south of the zenith
10	" "	for time and azimuth
17	" "	for azimuth, using a theodolite

At Memel, observations were made on

16	nights for	time and azimuth with meridian circle
13	" "	latitude
12	" "	azimuth, using a theodolite

At Königsburg, observations were made on

17 nights for latitude.

The work of calculation was made under rigorous least squares adjustment, and all errors of observation most carefully investigated. The application of least squares, in fact, was most fully developed by Bessel. In the triangulation computation there were 22 angle conditions and 9 side equations. The resulting latitudes and distances are:

	Latitudes	Distance between Parallels
Trunz	54 13 11.466″	
Königsberg	54 42 50.500″	28211.629 toises
Memel	55 43 40.446″	57965.346 "

LACAILLE'S ARC VERIFIED BY SIR THOMAS MACLEAR.

The results of Lacaille's work at the Cape of Good Hope, South Africa, in 1752, as previously shown, appeared large if the shape of the southern hemisphere of the earth were similar to that of the northern. The correctness of the early work could only be proved by a re-survey, and this was made during the years 1840–1848, under the direction of Sir Thomas Maclear, Royal Astronomer at the Cape.

Much time was spent and care taken to find the original stations used by Lacaille. They were nearly all identified, the triangulation stations being found by the charcoal remains of the signal fires, and also the probable location of the astronomical stations. These latter were identified from descriptions of the buildings which Lacaille had used, the foundations being discovered. As early as 1836 and 1837, minor triangulation around Cape Town had been executed, connecting the probable location of Lacaille's astronomical station with the then present site of the Royal Observatory, and the location of Sir John Herschel's reflecting telescope.

This triangulation depended upon two short measured bases, the longest being about 2900 feet, the other about 500 feet. These were measured by deal rods constructed by the surveyors themselves, and modeled after those used in England by General Roy. The latitude of Lacaille's probable station, resulting from using the determined position of the Royal

Observatory, gave S 33′ 55′ 17.11″. Lacaille, in his first report, gave it S 33′ 55′ 15″, and subsequently revised it, giving S 33′ 55′ 13.3″, and by a later revision S 33′ 55′ 12.6″. Von Zach in his discussion gives it as S 33′ 55′ 12.45″, while Prof. Henderson gives it as S 33′ 55′ 16.07″.

The Bradley sector. This instrument, used for obtaining the zenith distances of stars, was of the same type as the large Ramsden zenith sector, although made at an earlier date, probably about 1750. It had, however, been somewhat altered and remodeled from its original construction. In general, it consisted of a large triangular frame, carrying an inner frame with a long vertical spindle, resting in conical bearings. From this vertical spindle, at the end of horizontal arms, hung the telescope, 12½ feet long, aperture 3½ inches, and magnifying power of 70. The arc, of 12½ degrees, was graduated to 5 minutes by means of golden plugs inserted in a steel limb, and was used by means of the micrometer. The plumb-line, carrying a bob of 18 ounces suspended in a cup of water, was protected as far as possible from vibration due to air currents, by a case open on the front.

In observing, their method was first to point the telescope at the expected place of the star, and clamp the slide; then place the plumb-line, by means of its adjustments, directly over its mark and obtain the micrometer reading of the nearest graduation. At the instant of the star's passage, the micrometer reading was taken again, the difference of the two readings giving the amount to be added or subtracted from the nearest graduation mark. Collimation error was eliminated by using the face of the sector in one position one night, and in the reversed position the following night, and so on. Observations were made on stars culminating north and south of the zenith.

We may note here that instruments of this sort, depending upon a plumb-line for verticality, are subject to errors, due to the difficulty of bisecting perfectly the point over which the plumb-line passed, for in some cases it may hang too far away from the graduated arc, while in others it may touch and be slightly retarded. Also since it could not be reversed the same night, small errors of collimation might creep in.

Latitude observations near Lacaille's probable southern station at Cape Town. The actual position of Lacaille's station could not be used, owing to the fact that a building now covered its site, so the instrument was set up in a tent located in the yard of the dwelling a few feet south. Here observations were made commencing December 29th, 1837, and continued until February 19th, 1838. The location was not considered a good one because of the crowded quarters, and the frequent winds that swept through the yard. Hence, later in the year, a new position near there was selected. A guard-house was transformed into an observatory, giving better accommodations and the results obtained here were used in the final computations. Observations were also made at Table Mountain, in February and March, 1838, in order to determine the effect of the mountain on plumb-line deflection.

Observations at the northern end, Klyp Fonteyn, were made March 28th to April 21st, occupying what was first thought to be the original station, but later another point, that seemed to conform better to the description, was selected as being Lacaille's point. Minor triangulation connecting with the nearest station was done here, the work resting on a measured base of 1800 feet. In this work a steel chain was employed. This northern astronomical station of Maclear's was marked by burying in the ground to a depth of three feet, and resting in a hole in a rocky ledge, a quart bottle containing a memorandum of the party, work, etc. Over this was placed a flat stone and the hole was then filled in with dirt.

The resulting amplitudes, using the observations at the Guard House, Cape Town, are:

Amplitude from 20 stars north of the zenith	1° 13′ 14.163″
" " " " south " " "	1° 13′ 14.847″
Mean	1° 13′ 14.505″

Giving to each result a weight proportional to the quotient of the square of the number of observations of each star by twice the sums of the squares of the errors at both stations, there results for the amplitude from

Stars north of the zenith at Cape Town	1° 13′ 14.173″
" south " " " " " "	1° 13′ 14.958″

a difference of 0.78,″ the excess being on the southern stars. It is a remarkable fact that the observations of Lacaille gave an excess of 0.8″ on the southern stars.

This amplitude should be corrected for 261 feet, 216 at the northern point and 45 at the southern, or +2.56″, giving the amplitude as 1° 13′ 17.12″, as the result of 464 observations at Klyp Fonteyn and 669 at the Guard House, Cape Town. Lacaille's result was first published as 1° 13′ 17.33″ and after a recomputation as 1° 13″ 17.5″. These results checking so closely showed that there was no error in the astronomical work.

Base measurement. No definite location of the terminals of Lacaille's base could be found, although the small hills upon which they were situated were undoubtedly located. On this account a different position was selected, which was near, and which crossed that of Lacaille. The base apparatus used was that devised by Colonel Colby for the measurement of the Lough Foyle base. This apparatus was sent from England together with two standard iron rods, A and B, similar to those used by Colonel Colby. These rods were made under the direction of Sir George B. Airy, the Astronomer Royal.

The work was done in the same manner as in the Lough Foyle base, Sir Thomas Maclear being assisted by details of soldiers sent from both England and the Cape. During the measurement, the bars were compared with the standards by means of beam compasses. One serious accident occurred during the work, when a whirlwind upset the measuring bars, blowing bars D and E to the ground and permanently rendering bar E unfit for further use. The measurement was checked by dividing the base into sections and computing their length from special triangulation. The average temperature of the measurement was 80.16° Fah. The resulting length corrected, and reduced to sea level at a temperature of 62° Fah., and referred to the Ordnance standard bar 0, was 42,819.065 feet.

Instruments and signals. The horizontal angles were measured by the use of two theodolites. One was made by Thomas Jones and was similar to the Ramsden theodolite. It had a 20 inch circle, was graduated to 10 minutes, had three micrometer microscopes, and by means of these read to single seconds. The other was a repeating theodolite made by Reichenbach and Ertel, having an 8½ inch circle, graduated to 10 minutes, and reading to 10 seconds by means of four microscopes.

The signals were for the most part reflectors or heliostats, but on some of the short lines under 20 miles wooden signals were used. Most of the stations were occupied directly but a few eccentrically. The work was extended both to the north and south of that of Lacaille, embracing

64 primary triangles and 52 secondary ones. Astronomical azimuths were determined at three points and the whole work was adjusted by least squares.

The following table gives the deduced results:—

Station	Astronomic latitude	Geodetic latitude	Distance between parallels	Computed distance
North End	29° 44′ 17.69″	29° 44′ 17.32″	feet	feet
			224600.6	224600.7
Kamies Sector Berg.	30 21 29.06	30 21 20.70		
			811506.8	811506.7
Heerenlogement Berg.	31 58 9.03	31 58 9.64		
			1526385.1	1526385.0
Royal Observatory.	33 56 3.20	33 56 3.20		
			1632581.4	1632581.4
Zwart Kop.	34 13 32.12	34 13 33.80		
			1678373.8	1678374.1
Cape Point.	34 21 6.26	34 21 6.81		

The geocentric latitudes were computed by the use of Airy's elements. The computed distances between the parallels were obtained by using Bessel's formula

$$S=a(l-n)^2(l+n)\left\{\begin{array}{l} m\phi-m_1\sin\phi\cos 2\lambda+m_2\sin 2\phi\cos 4\lambda \\ -m_3\sin 3\phi\cos 6\lambda \text{ — — — — —}\end{array}\right.$$

The latitude of the Royal Observatory was taken as the starting point. The result of Lacaille's distance between his terminal points was 445,505.5 feet and that obtained from this work, 445,361.5, or an excess in Lacaille's work of 144 feet. This means that his base was in excess 13.586 feet or 2.1245 toises. Even this error, however, does not help to rectify the supposed error. These results show that Lacaille's work was very well done. Computing, however, the position of his northern station by the elements of the earth, there is found to be a discrepancy of 8.55″, which is due to the deflection of the plumb-line, and explains why Lacaille's results were large.

THE EARTH CONSIDERED AS A SPHEROID.

While the earth was definitely recognized as an oblate spheroid as a result of Maupertuis's work in 1734, no general computation from considering a number of arcs collectively was made until the beginning of the nineteenth century. Tables had been made, however, which allowed for the increase in the length of a degree as the latitude increased. Bougeur, for instance, published such tables, in which he assumed the increase to vary as the fourth power of the latitude.

In 1819, Walbeck (9, p. 183), in a publication entitled "Dessertation de forma et magnitudina telluris, ex dimensis arcubus meridiani definiendis," began the outline of the correct treatment. He considered six arcs, the Peruvian, the two Indian, the French, English and the Swedish arc of Svanberg. His method was that of least squares although not fully developed He laid down the fundamental principle, that the squares of the errors between the measured and computed amplitudes should be a minimum. As a result of his work he obtained for the ellipticity $\frac{1}{302.78}$ and for the average value of the length of 1 degree 57,009.758 toises. We may note in considering amplitude alone that he neglects all determined points lying between the terminals, and hence does not obtain the most accurate results.

Laplace. In the Méchanique Céleste of Laplace published in 1822, (83, Vol. p. 443-63) a discussion is given of the ellipticity of the earth resulting from using the seven arcs, the Peruvian, Lacaille's, Mason and Dixon's, Boscovich's, the French, Liesganig's and the Lapland arc of Maupertuis. The length of a degree on the elliptical hypothesis is taken in the form $a=z-py$, which really corresponds to the formula for the length of the seconds pendulum, or p is taken as $\sin\phi$.

Considering small errors to occur, the form of the equation will be $a-z-py=x$, and for each arc an equation of this form will occur. The solution of these condition equations gave the ellipticity as $\frac{1}{312}$, and the length of a degree as 1 degree = 51,077.70 toises + 493.86 $\sin^2\phi$. No values for the semi-axes were given. As several of these arcs have since been found to be in error the results of the discussion are practically worthless.

Schmidt. (9. pp. 192-202). In 1829, Dr. Schmidt extended the work outlined by Walbeck. To the six arcs discussed by Walbeck was added the Hannoverian arc. He improved the treatment of least squares by bringing into the discussion not only the terminal points, but the intermediary points as well. Thus he considered seven arcs with the 25 determined points. As a result of his work he obtained the ellipticity as $\frac{1}{298.32}$, the value of the average degree of the meridian 57,008.662 toises, for the semi-axes a=3,271,837.5 toises, b=3,260,920.3 toises and the meridian quadrant as 5,130,779.58 toises.

Everest's determination of 1830. In Colonel Everest's report of the two Indian arcs, published in 1830, he gives a discussion of the seven following arcs

Arc	Amplitude	Length in Eng. feet	Middle latitude
1. Pannea to Damargida	9° 53′ 43.57″	3591779.04	13° 6′ 24.29″ N.
2. Damargida to Kalianpur	6 3 55.78	2202819.54	21 5 13.86 N.
3. Pannae to Kalianpur	15 57 39.34	5794598.58	16 8 22.18 N.
4. Cotchesqui to Tarqui	3 7 3.12	1131056.34	1 31 00.34 S.
5. Formentera to Greenwich	12 48 42.85	4670810.33	45 4 17.54 N.
6. Mallorn to Pahtavara	1 37 19.57	593277.39	60 20 10.05 N.
7. Dunnose to Clifton	2 50 23.38	1036409.5	52 2 19.90 N.

In computing he used the following formula (73, Vol.2, page 126:)

$$S=al-(a-b)\tfrac{1}{2}\,[\,1+3\cos 2\lambda \sin 1\,] + \frac{(a-b)^2}{a}\,\frac{1}{16}\left\{1+\frac{15}{2}\cos 4\lambda \sin 2\lambda\right\}$$

where s= length, and l=the amplitude in seconds of the arc, and a and b= the semi-axes of spheroid. He adopted the following method for determining the value of a and b from the data of a pair of arcs of different parallels. If m, n, and p represent the different coefficients of a, $(a-b)$, and $(\frac{a-b}{a})^2$ in the above equation for an arc whose length is t; and if μ, ν, and π denote the corresponding coefficients for another arc whose length is τ which is situated in a higher or lower latitude than the former, then putting $c=a-b$ the two following equations are obtained

$$t=ma+nc+p\frac{c^2}{a}$$

$$\tau=\mu a+\nu c+\pi\frac{c^2}{a}$$

and if c' and a' are approximate values of a and c found by neglecting the last terms of these equations so that $a=a'+\delta a'$ and $c=c'+\delta c'$ then neglecting the products of $\delta a'$ and $\delta c'$

$$a'=\frac{\nu t-n\tau}{m\nu-\mu n} \text{ and } \delta a'=\frac{c'^2(n\pi-\nu p)}{(m\nu-\mu n)\,a'-(m\pi-\mu p)2c'}$$

$$c'=\frac{\mu t-m\tau}{m\nu-\mu n} \text{ and } \delta c'=\frac{c'^2(m\pi-\mu p)}{(m\nu-\mu n)\,a'-(m\pi-\mu p)\,2c'}$$

From the use of this method in comparing any two arcs there results the following equations:

Arc employed	a ft.	b ft.	$a-b$ ft.	$\frac{a-b}{a}$
1 and 2	20 889 781.	20 839 631.	50 151	$\frac{1}{416\cdot55}$
3 " 4	20 901 599.	20 844 246.	57 353	$\frac{1}{364\cdot44}$
3 " 5	20 922 932.	20 853 375.	69 557	$\frac{1}{300\cdot80}$
3 " 6	20 920 435.	20 852 301.	68 134	$\frac{1}{307\cdot05}$
3 " 7	20 919 718.	20 851 994.	67 724	$\frac{1}{308\cdot01}$

Everest considered the comparison of small arcs to be worth but little, and thus selected for the best values of a and b those given by the comparison of arcs 3 and 5, neglecting all the others.

His results were then— $\begin{cases} a=20\ 922\ 932. \text{ feet} \\ b=20\ 853\ 375. \text{ ''} \\ c=\frac{1}{300.80} \quad \text{ ''} \end{cases}$

These are known as "Everest's 1st constants," and have been used since that time, in India, in all calculations in which the figure of the earth enters.

Everest's determination of 1847. For this determination Everest considered the following arcs:

Arc			Given by
1. Kalianpur	to	Kaliana	Colonel Everest
2. Damargida	"	Kalianpur	" "
3. Damargida	"	Kaliana	" "
4. Damargida	"	Punnae	Colonel Lambton
5. Punnae	"	Kaliana	Colonels Lambton and Everest
6. Formentera	"	Dunkirk	Delambre, and others
7. Jacobstadt	"	Hochland	Prof. Struve
8. Tarqui	"	Cotchesqui	Bougeur and De la Condamine
9. Dunnose	"	Clifton	General Mudge
10. Pahtavara	"	Mallorn	Svanberg
11. Jacobstadt	"	Dorpart	Prof. Struve
12. Dorpat	"	Hochland	" "

The comparisons were made in nearly the same manner as in his first consideration, but instead of accepting, as he did there, the results from the longest arc and neglecting the others, they were combined together with weights deduced from the comparison of the arcs with one another.

The results were— $\begin{cases} a=20{,}920{,}902. \text{ feet} \\ b=20{,}853{,}642. \text{ ''} \\ \dfrac{a-b}{a}=\frac{1}{311.04} \quad \text{ ''} \end{cases}$

These are known as "Everest's 2d Constants," but they have never been employed in India.

Airy. In 1830, the Astronomer Royal of England, Sir George B. Airy, published his investigation of 14 meridian arcs. The theoretic discussion governing his treatment is briefly as follows: Considering the earth to be an ellipsoid of rotation, the radius of curvature at any point of a meridian section can be shown by the calculus to be:

$\rho = \dfrac{a^2 b^2}{(b^2 \sin^2 L - a^2 \cos^2 L)^{\frac{3}{2}}}$ where a and b are the semi-major and semi-minor axes of the ellipse and L the latitude of the point. Calling $e = \dfrac{a-b}{b}$ (which is different from the present notation) and neglecting the square of e, the above may be put in the following form: $\rho = b\,(1 - e + 3e \sin^2 L)$. The length of a portion of meridian arc will then be

$$ds = \rho\, dL \quad \text{or} \quad ds = b dL\,(1 - e + 3e \sin^2 L)$$

$$= b dL\,(1 + \frac{e}{2} - \frac{3e}{2} \cos 2L)$$

Hence on integrating $S = b\,[\,(1 + \frac{e}{2})L - \frac{3e}{4} \sin 2L\,]$ which represents the length of the meridian arc from the equator to a point in latitude L. Replacing L by λ' and λ, the latitudes at two different points, and taking the difference of the results, the value of the arcs between them is

$$\text{arc} = b\left[\left(1 + \frac{e}{2}\right)(\lambda' - \lambda) - \frac{3e}{4}\,(\sin 2\lambda' - \sin 2\lambda)\right]$$

$$= b\left[\left(1 + \frac{e}{2}\right)(\lambda' - \lambda) - \frac{3e}{2} \cos(\lambda' + \lambda) \sin(\lambda' - \lambda)\right]$$

$$= b\left[\ 1 + \frac{e}{2} - \frac{3e}{2} \cos(\lambda' + \lambda) \frac{\sin(\lambda' - \lambda)}{\lambda' - \lambda}\right](\lambda' - \lambda)$$

$$\underset{\text{(in seconds)}}{\text{arc}} = b\left[1 + \frac{e}{2} - \frac{3e}{2} \cos(\lambda' + \lambda) \frac{\sin(\lambda' - \lambda)}{\lambda' - \lambda}\right](\lambda' - \lambda) \sin 1''$$

The following table contains the arcs as given and discussed. The results are in English feet, the relation of the English foot to the French units being based upon the standard of Shuckburgh. These relations are, 1 toise equals 6.394596 Eng. feet, and 1 metre equals 3.280899 Eng. feet.

Arc.	Latitude of middle point.	Amplitude.	Length in Eng. feet.
1. Peruvian arc as calculated by Delambre	1° 31′ 00″	3° 07′ 3.1″	1,131,057
2. Maupertuis's Swedish arc	66 19 37	0 57 30.4	351,832
3. French arc by Lacaille and Cassini de Thury	46 52 02	8 20 00.3	3,040,605
4. Roman arc by Boscovich	42 59 00	2 09 47.0	787,919
5. Lacaille's arc near the Cape of Good Hope	33 18 30	1 13 17.5	445,506
6. American arc by Mason and Dixon	39 12 00	1 28 45.0	538,100
7. French arc from Formentera to Dunkirk	44 51 02	12 22 12.6	4,509,402
8. Svanberg's Swedish arc	66 20 10	1 37 19.3	593,278
9. English arc from Dunnose to Burleigh Moor	52 35 45	3 57 13.1	1,442,953
10. Lambton's lst Indian arc	12 32 21	1 34 56.4	574,368
11. Lambton's 2nd English arc as extended by Everest	16 8 22	15 57 40.2	5,794,599
12. Piedmontese arc by Plana and Carlini	44 57 30	1 07 31.1	414,657
13. Hanoverian arc by Gauss	52 32 17	2 00 57 4	736,426
14. Russian arc by Struve	58 17 37	3 35 05.2	1,309,742

Using the given formula for the length of the arc, there result 14 equations; from the first arc $1131057 = .0544111\,b - 0.05426be$, and others similar for the remaining thirteen. Comparing these arcs, two by two, it was found that the greatest variations in e occurred in those arcs which were in mountainous country. This indicated that local attraction influenced the observations and led to the discarding of arcs number 1, 2, 4, 5, 8, and 12. From the remaining arcs the earth's figure was determined. The method employed was not that of least squares; concerning this method Airy makes the following statement (6, p. 219), "In determining elements from a number of observations or measures, the method of minimum squares has frequently been used. We have rejected the use of this method for the following reason: It is perfectly certain that the elements determined by this method, if substituted in the equations of conditions, will generally give the greatest apparent errors of linear measure in the smallest arc, and vice versa. A consequence so opposed to common sense cannot, we think, be supported by any symbolical reasoning. The doctrine of chances (from which this method is deduced) is more liable than any other to errors of omission in the preliminary considerations for the solution of any problem: and we prefer resting in the belief that there is some such error in the proof of this method, to receiving the consequence above mentioned. We have, therefore, thought best to use the method commonly employed in astronomy, viz., to take the sum of groups of the equations of conditions, and to consider each sum as one equation; the groups being selected so as to make the coefficient of e large and positive in one case and large and negative in the other." Adopting this method and comparing the sums of arcs numbers 3, 6, 7, 9, 13, and 14, with those of numbers 10 and 11, there results $b = 20,854,270$ and $e = .0033808$. Comparing the sums of numbers 7, 9, 13, and 14, with the sum of numbers 10 and 11, there results: $b = 20,853,355$ and $e = .0033232$. Taking the mean as the better value of each we have $b = 20,853,810$ feet and $e = .0033520$ feet.

Using these results to compare the lengths at the various middle latitudes, it was found that the resulting errors in amplitude were for the most part less than 5 seconds, only five arcs giving over that. The arcs agreeing most nearly were numbers 8, 10, 11, and 14, while arc number 12 was the greatest in error, 43.4 seconds, a much larger error than that given by any other. Considering the earth an ellipsoid of revolution, the resulting dimensions are:

a, semi-equatorial axis, $= 20{,}923{,}713$ feet $= 3{,}962.824$ miles
b, " polar " , $= 20{,}853{,}810$ " $= 3{,}949.585$ "
ratio of a to b as 299.33 to 298.33,

ellipticity $= \dfrac{a-b}{b} = \dfrac{1}{298.33} = .003{,}352.$

The present designation of ellipticity would be

$$\frac{a-b}{a} = \frac{1}{299.33}$$

Bessel in 1837 gave a rigorous discussion of the determination of the semi-axes of the earth considered as a spheroid of revolution. His method briefly is as follows: Assuming the earth to be an oblate spheroid of revolution, the radius of curvature at any point in latitude ϕ can be shown by the differential calculus to be $R = \dfrac{a\,(1-e^2)}{(1-e\sin^2\phi)^{\frac{3}{2}}}$ where

$e = \dfrac{\sqrt{a^2-b^2}}{a},$ the eccentricity.

The length of any meridian distance is therefore then,

$$s = a\,(1-e^2)\int \frac{d\phi}{(1-e\sin^2\phi)^{\frac{3}{2}}}$$

Expanding this into series and integrating term by term, there can be obtained

$$s = a(1-n)^2(1+n)\,N\left\{\phi - \alpha\sin 2\phi + \frac{1}{2}\alpha'\sin 4\phi - \frac{1}{3}\alpha''\sin 6\phi\right\}$$

in which $n = \dfrac{a-b}{a+b},$ and $N, \alpha, \alpha', \alpha''$, etc., are functions of n.
Calling g the mean length of a degree of the meridian the value of the above for $\phi = 180°$ will be

$$180\,g = a\,(1-n)^2\,(1+n)\,N\pi$$

Hence,

$$s = \frac{180g}{\pi}\left\{\phi - \alpha\sin 2\phi + \frac{1}{2}\alpha'\sin 4\phi - \frac{1}{3}\alpha''\sin 6\phi + \ldots\right\}$$

The difference between two latitudes ϕ' and ϕ will then be

$$s' - s = \frac{180\,g}{\pi}\left\{ \begin{array}{l} \phi' - \phi - 2\alpha \sin(\phi' - \phi)\cos(\phi' - \phi) \\ + \alpha' \sin 2(\phi' - \phi)\cos 2\,(\phi' - \phi) - - \end{array} \right\}$$

on substituting for the function of $2\,\phi$ its value in terms of ϕ. If a number of arcs are taken, it is not probable that all will exactly satisfy the same spheroid, hence to each observed latitude should be applied a small correction x, the sums of the squares of which should be a minimum in order to obtain the spheroid which will agree to the best advantage with all the arcs. In the above, then, substitute for ϕ', $\phi' + x'$, and for ϕ, $\phi + x$. As a result of this and a rather long process of transformation, an equation of the following form is obtained; $x'_1 - x_1 = m + ai + bk$, in which x_1 is the correction at the southern extremity, x'_1 that at the northern point, while m, a, and b are functions of observed quantities and hence can be computed, and i and k represent quantities to be determined in the process of the work. For every arc or portion of arc an equation of the above form will ensue. The equation can be written $x'_1 = x_1 + m + ai + bk$, and if there are several parts to the arc, the equation will take the form

$$\begin{aligned} x'_1 &= x_1 + m_1 + a_1 i + b_1 k \\ x''_1 &= x_1 + m'_1 + a'_1 i + b'_1 k \\ &\text{etc.} \end{aligned}$$

The sum of the squares of these errors which may be represented by

$$\begin{aligned} U = x_1{}^2 &+ (m_1 + a_1 i + b_1 k)^2 \\ &+ (m'_1 + a'_1 i + b'_1 k)^2 \\ &+ (\quad \text{etc.} \quad)^2 \\ + x_2{}^2 &+ (m_2 + a_2 i + b_2 k)^2 \\ &+ (m'_2 + a'_2 i + b'_2 k)^2 \\ &+ (\quad \text{etc.} \quad)^2 \end{aligned}$$

must be equal to zero.

Hence, $\frac{\delta U}{\delta i} = 0$, $\frac{\delta U}{\delta k} = 0$, $\frac{\delta U}{\delta x} = 0$, $\frac{\delta U}{\delta x_2} = 0$,

$\frac{\delta U}{\delta x_3} = 0$, $\frac{\delta U}{\delta x_4} = 0$, $\frac{\delta U}{\delta x} = 0$, etc.

all the above being understood to be partial derivatives, giving as many equations as unknown quantities, from which the value of these unknown quantities can be determined by algebraic methods. The following are the data used by Bessel in his discussion:

Arc.	Latitude points.	Latitudes.	Amplitudes.	Dist. between parallels.
1. Peru	Tarqui	3°04′32.068″		
	Cotchesqui	0 02 31.377	3° 7′ 3.455″	176875.5 t.
2. 1st. East Indian	Trivande-porum	11 44 52.590		
	Paudree	13 19 49.018	1 34 56.428	89813.01 t.
3. 2nd. East Indian	Punnae	8 09 31.132		
	Putchapol-lian	10 59 42.276	2 50 11.144	160944.20 t.
	Dodagoontah	12 59 52.165	4 50 21.033	274694.30 t.
	Namthabad	15 05 53.562	6 56 22.430	393828.09 t.
	Daumergida	18 03 16.245	9 53 45.113	561690.06 t.
	Takal K'hera	21 05 51.532	12 56 20.400	734570.43 t.
	Kalianpur	24 07 11.860	15 57 40.728	906171.67 t.
4. French	Formentera	38 39 56.11		
	Montjouy	41 21 44.96	2 41 48.85	153605.77 t.
	Barcelona	41 22 47.90	2 42 51.79	154548.9 t.
	Carcassonne	43 12 54.30	4 32 58.19	259104.8 t.
	Evaux	46 10 42.58	7 30 46.43	427951.5 t.
	Pantheon	48 50 49.37	10 10 53.26	580244.6 t.
	Dunkirk	51 02 08.85	12 22 12.74	705189.4 t.
5. English	Dunnose	50 37 07.633		
	Greenwich	51 28 39.000	0 51 31.367	49059.89 t.
	Blenheim	51 50 27.652	1 13 19.999	69829.19 t.
	Arburyhill	52 13 28.031	1 36 20.398	91696.39 t.
	Clifton	53 27 31.130	2 50 23.497	162075.93 t.
6. Hanno-verian	Gottingen	51 31 47.85		
	Altoona	53 32 45.27	2 00 57.42	115163.725 t.
7. Danish	Lauenburg	53 22 17.046		
	Lysabbel	54 54 10.352	1 31 53.306	87436.538 t.

Arc.	Latitude points.	Latitudes.	Amplitudes.	Dist. between parallels.
8. Prussian	Trunz	54 13 11.466		
	Königberg	54 42 50.500	0 29 39.034	28211.629 t
	Memel	55 43 40.446	1 30 28.980	86176.976 t.
9. Russian	Belin	52 02 40.864		
	Nemesch	54 39 04.519	2 36 23.655	148811.418 t.
	Jacobstadt	56 30 04.562	4 27 23.698	254543.454 t.
	Bristen	56 34 51.550	4 32 10.686	259110.085 t.
	Dorpat	58 22 47.280	6 20 6.416	361824.461 t.
	Hochland	60 05 09.771	8 02 28.907	459363.008 t.
10. Swedish	Malorn	65 31 30.265		
	Pahtavara	67 08 49.830	1 37 19.565	92777.981 t.

If we compare these results with those previously given under their respective accounts, it will be found that they do not agree in all cases. The reason for this is explained by the fact that Bessel has carefully investigated all, and taken what he believed to be the more correct values. For instance, on arc number 1, the Peru arc, he has taken the mean between the results given by Delambre, in the base of the Metric System, and those of Von Zach. Also in the 2d arc, a slight correction has been made from knowing more accurately the relation between the fathom and the toise, using 1 toise equal to 1.06576542 fathoms. In the 3rd arc, he has rejected the observed zenith sector readings for the determination of the latitudes, and made a new series of calculations for them. The 4th arc is taken directly from the base of the Metric System, by Delambre. The 5th arc has been corrected, as in the second, for the new relation between the toise and fathom. The others have been taken directly from their respective publications.

As a result the following determinations were obtained:

$$\text{semi-axes} \begin{cases} a = 3{,}271{,}953.854 \text{ toises} & \log. a = 6.514{,}807{,}169{,}9 \\ b = 3{,}261{,}072.900 \quad \text{"} & \log. b = 6.513{,}507{,}360{,}3 \end{cases}$$

g = mean value of $1^\circ = 570{,}011.453 \pm 2.900$ toises

$$\text{length of quadrant} = \frac{90\ (g)\quad 864}{443.296} = 10{,}000{,}565.278 \text{ metres.}$$

Later it was found that the French arc was in error and, in 1841, Bessel carefully examined this arc and made a complete and adjusted recomputation, by the method of least squares, obtaining for the distance between Montjouy and Formentera, 153,673.610 toises, very nearly the mean of recomputations made in June, 1841, by Puissant and others.

Bessel then made a recomputation of the dimensions of the spheroid, using all his former data except the French arc, for which he used the following:

Arc.	Latitude points.	Latitudes.	Amplitudes.	Dist. between parallels.
4. French	Formentera	38°39′56.11″		
	Montjouy	41 21 44.96	2 41 48.85	153673.61 t.
	Barcelona	41 22 47.90	2 42 51.79	154616.74 t.
	Carcassonne	43 12 54.30	4 32 58.19	259172.61 t.
	Evaux	46 10 42.58	7 30 46.43	428019.30 t.
	Pantheon	48 50 49.37	10 10 53.26	580312.41 t.
	Dunkirk	51 02 08.85	12 22 12.74	705257.21 t.

Computations were made, as in the former case, giving the following results:

a = 3,272,077.14 toises   log. a = 6.514,823,5
b = 3,261,139.33 "   " b = 6..513,369,3
g = 57,013.109 ± 2.840,3 toises
quadrant = 5,131,179.81 toises = 10,000,855.76 ± 498.23 metres.

Clarke's spheroid. The work of Captain, and later General A. R. Clarke, of the British Ordnance Survey, stands preeminent in matters pertaining to the figure of the earth. In 1856 (75, pp. 607-626), was published his discussion of 10 arcs, these being the same that Bessel had used, except that in the English and Indian arcs better and more correct positions were used. The results were:

a = 20,924,933 feet of ordnance standard O_1
b = 20,854,731 " " " " O_1
$e = 1/294.26$

A little later, in 1858, was published a more complete account of his investigations (76). Nine arcs having 67 latitude stations were here discussed, the Lapland arc being omitted.

$$\text{The results were:} \begin{cases} a = 20{,}926{,}348 \text{ feet} \\ b = 20{,}855{,}223 \quad \text{“} \\ e = 1/294.26 \end{cases}$$

His method of treatment was by least squares and differed from that used by Bessel in only minor points. Hence we may omit the theoretical considerations.

Again, in 1866, was published a discussion of five arcs, the results of which are known as Clarke's elements, and are in general use to-day in computations depending on the figure of the earth. In order that the results should be as accurate as possible, the different governments were asked to send their respective standards of length to England, that they might be compared directly with the English yard. This would enable all modern measured arcs (since as a rule they depended on governmental standards) to be reduced to the same standard. The standards sent were very carefully compared, and the results are shown by the following table (77, pp. 280):

Standard.	At stand. temp. Fah.	Expressed in terms the standard yard.	Expressed in inches.	Expressed in lines of the toise $L = T/864$	Expressed in milli-metres.
The yard		1.00000000	36.000000	405.34622	914.39180
Copy NO.55 of the yard	62	0.99999960	35.999986	405.34606	914.39143
Ordnance standard ft.	62	0.33333284	11.999982	135.11521	304.79681
Indian standard foot	62	0.33333611	12.000100	135.11653	304.79980
Ordnance 10 foot bar O_1	62	3.33333717	120.000138	1351.15563	3047.97616
Ordnance 10 foot bar $O1_1$	62	3.33335432	120.000755	1351.16259	3047.99184
Indian 10 foot bar 1_s	62	3.33340138	120.002450	1351.18166	3048.03488

Standard.	At stand. temp. Fah.	Expressed in terms the standard yard.	Expressed in inches.	Expressed in lines of the toise $L = T/864$	Expressed in milli-metres.
Indian 10 foot bar 1_B	62	3.33353284	120.007182	1351.23495	3048.15508
Indian 10 foot bar 1_b	62	3.33331457	119.999324	1351.14647	3047.95550
Austrailan standard $O1_4$	62	3.33330427	119.998954	1351.14230	3047.94608
Australian standard $O1_5$	62	3.33333747	120.000149	1351.15576	3047.97644
Ordnance toise	61.25	2.13166458	76.739925	864.06219	1949.17660
Ordnance metre	61.25	1.09374800	39.374928	443.34662	1000.11420
Royal Soc. metre àtraits	32.0	1.09360478	39.369772	443.28857	999.98324
Prussian toise No.10	61.25	2.13150911	76.734328	863.99917	1949.03444
Belgian toise No.11	61.25	2.13150851	76.734306	863.99893	1949.03390
Russian double toise	61.25	4.26300798	153.468287	1727.99419	3898.05952
The toise		2.13151116	76.734402	864.00000	1949.03632
The metre		1.09362311	39.370432	443.29600	1000.00000

It will be noticed from the inspection of the table, that no direct comparison was made with the standard metre of France, but that its value was ascertained by indirect comparison. It has since been found that the relation between the standard yard and metre here given is widely in error. The best now known relation is 1 metre = 1.0936143 yards
= 39.370,113 inches

The data of the six arcs used in computation by Clarke are as follows:

FRENCH ARC.

Stations.	Astronomical latitude.	Distance of parallels in toises.	Distance of parallels in stand. feet.
Formentera	38° 39′ 53.17″		
Montjouy	41 21 44.96	153,673.61	982,671.04
Barcelona	41 22 47.90	154,616.74	988,701.92
Carcassonne	43 12 54.30	259,172.61	1,657,287.93
Pantheon	48 50 47.98	580,312.41	3,710,827.13
Dunkirk	51 02 08.41	705,257.21	4,509,790.84

The latitudes of the stations were taken from corrected results of the French astronomers excepting at Dunkirk, which was the value obtained by the use of the Ramsden zenith sector.

ENGLISH ARC

Stations.	Astronomical latitude.	Distance of parallels in feet of O_1 from Greenwich.	Distance of parallels from Fomentera in stand. feet.
Formentera			
Greenwich	51° 28′ 38.30″		4,671,198.3
Asbury	52 13 26.59	272,639.0	4,943,837.6
Clifton	53 27 29.50	722,864.3	5,394,063.4
Kellie Law	56 14 53.60	1,742,021.4	6,413,221.7
Stirling	57 27 49.12	2,186,122.5	6,857,323.3
Saxavord	60 49 37.21	3,415,618.5	8,086,820.7

In this, the latitude at Saxavord was the mean of the observed and the results of two other stations near by reduced to Saxavord.

RUSSIAN ARC.

Stations.	Astronomical latitude.	Distance of parallels in toise.	Distance of parallels in standard feet.
Staro-Nebrassowka	45° 20′ 02.94″		
Wodolui	47 01 24.98	96,415.136	616,529.81
Ssuprunkowzi	48 45 03.04	194,973.124	1,246,762.17
Kremenetz	50 05 49.95	271,724.510	1,737,551.48
Belin	52 02 42.16	382,943.521	2,448,745.17
Nemesch	54 39 04.16	531,753.042	3,400,312.63
Jacobstadt	56 30 04.97	637,483.921	4,076,412.28
Dorpat	58 22 47.56	744,764.484	4,762,421.43
Hogland	60 05 09.84	842,303.102	5,386,135.39
Kilpi-maki	62 38 05.25	988,016.669	6,317,905.67
Tornea	65 49 44.57	1,170,810.973	7,486,789.97
Stuor-oivi	68 40 58.40	1,334,032.877	8,530,517.90
Fuglenaes	70 40 11.23	1,447,786.783	9,257,921.06

SOUTH AFRICAN ARC BY MACLEAR.

North End	29° 44′ 17.66″		
Heerenlogement	31 58 9.11	811,506.8	811,507.7
Berg			
Royal Observatory	33 56 3.20	1,526,385.1	1,526,386.8
Zwart Kop	34 13 32.13	1,632,581.4	1,632,583.3
Cape Point	34 21 6.26	1,678,373.8	1,678,375.7

INDIAN ARC.

Station.	Astronomical latitude.	Distance of parallels in feet of O_1.	Distance of parallels in standard feet.
Pannae	8° 09′ 31.132″		
Putchapollian	10 59 42.276	1,029,173.7	1,029,174.9
Dodagoontah	12 59 52.165	1,756,560.0	1,756,562.0
Namthabad	15 05 53.562	2,518,373.4	2,518,376.3
Daumergida	18 03 15.292	3,591,784.3	3,591,788.4
Takalkhera	21 05 51.532	4,697,324.1	4,697,329.5
Kalianpur	24 07 11.262	5,794,689.0	5,794,695.7
Kaliana	29 30 48.322	7,755,827.0	7,755,835.9

PERUVIAN ARC.

Station.	Astronomical latitude.	Distance of parallels in toises.	Distance of parallels in standard feet.
Tarqui	S 3° 04′ 32.068″		
Cotchesqui	N 0 02 31.387	176,875.5	1,131,036.3

From these six arcs and forty-one latitude stations, there results forty equations of corrections, which upon solution by least squares give, considering the earth as a spheroid of revolution, the following

Semi-axes	Feet	Length in Toises	Metres
Equatorial semi-axis, a,	20,926,062	3,272,492.3	6,378,206.4
Polar semi-axis, b,	20,855,121	3,261,398.4	6,356,583.8

$$e = \frac{a-b}{a} \qquad 1/294.98$$

SUMMARY OF THE EARTH CONSIDERED AS A SPHEROID.

Year by whom.	Value of equatorial semi-axis a		Value of polar semi-axis b		Ellipticity.	Length of meridian quadrant.	
1819 Walbeck					1/302.78	10000268	m
1822 Laplace	— — — —		— — —		1/312.0		
1829 Schmidt	3271837.5	t.	3260920.3	t.			
	6376835	$m.$	6355558	$m.$	1/298.32	10000008	$m.$
1830 Everest	20922932	ft.	20853375	ft.		— — — —	
	6377303	$m.$	6356103	$m.$	1/300.8	— — — —	
1830 Airy	20923713	ft.	20853810	ft.			
	6377440	$m.$	6356234	$m.$	1/299.33	10000976	$m.$
1837 Bessel	3271953.8	t.	3261072.9	t.			
	6377063	$m.$	6355770	$m.$	— —	10000565	$m.$
1841 Bessel	3272077.1	t.	3261139.3	t.			
	6377282	$m.$	6355984	$m.$	1/299.2	10000856	$m.$
1847 Everest	20920902	ft.	20853642	ft.			
	6375685	$m.$	6356183	$m.$	1/311.04	— — — —	
1856 Clark	20924933	ft.	20854731	ft.			
	6377913	$m.$	6356515	$m.$	1/298.07	10001515	$m.$
1858 Clark	20926348	ft.	20855223	ft.			
	6378360	$m.$	6356665	$m.$	1/294.26	— — — —	
1866 Clark	20926062	ft.	20855121	ft.			
	6378257	$m.$	6356634	$m.$	1/294.98	10001847	$m.$

In the reductions, in the above and following tables, in order to change the original measures to metres, the following relations have been used:

1 toise = 1.949 03 metres
1 yard = .914 399 2 "
1 metre = 1.093 614 3 yards = 39.370 111 3 inches
1 stadia = 202½ English yards.

SUMMARY OF ARC

Date of measurement.	Measurement made by.	Length.		Amplitude.
230 B. C.	Eratosthenes in Egypt	5000	stadia	7° 12′
100 B. C.	Posidonius in Egypt and Rhodes	5000	"	7 30
827 A. D.	Al Mamum in Arabia	56⅔	Ar. ml.	1 00
1528	Fernal in France	57746	t.	1 00
1617	Snell in Holland	33930	per.	1 11 30″
1633–1635	Norwood in England	9149	chs.	2 28 00
1658	Riccoli and Girmaldi in Lombardy	20439	paces	19 25
1669	Picard in France	78907	t.	1 22 55
1681–1718	Cassini in France	360614	t.	6 18 57
		125454	t.	2 12 09.3
1736	Maupertuis and others in Lapland	55023.5	t.	57 28.67
1735–1739	De la Condamine and others in Peru	176958.44	t.	3 07 01.0
1734–1742	Cassini	125431	t.	2 11 50.28
		99990	t.	1 45 07.33
		15767	t.	2 43 51.5
		94308	t.	1 39 11.2
1750–1751	Marie and Boscovich in Italy	161253.6	paces	2 09 47
1752	Lacaille in South Africa	69669.1	t.	1 13 17.5

MEASUREMENTS.

Middle latitude.		Length in original measure.		Length in metres.	Length of 1° based on Clarke's spheroid of 1866.	M.	Measuring apparatus used.	
	Value of 1 degree the earth considered as a sphere							
N. 28°	app.	700 stadia		129456	110816	"	Unknown	
N. 35	"	666⅔ "		123292	110939	"	"	
N. 33	"	56⅔ A. M.		---	---	"	"	
N. 49 20	"	57746	t.	112547	111217	"	Rolling wheel	
N. 52 00	"	55100	t.	107390	111269	"	Wooden rods	
N. 52 49	"	367196	ft.	111921	111285	"	Chain	
N. 44 34	"	61478	t.	119821	111124	"	Wooden Rods	
N. 49 10	"	57060	t.	111210	111214	"	"	"
N. 45 40 44″		57097	t.	111282	111145	"	"	"
N. 49 56 17		56960	t.	111015	111210	"	"	"
N. 66 19 37		57437.9	t.	111945	111520	"	"	"
S. 1 31		56749	t.	110604	110569	"	"	"
N. 49 56 07		57084	t.	111257	111210	"	"	"
N. 47 57 38		57071	t.	111232	111190	"	"	"
N. 45 43 08		57040	t.	111172	111145	"	"	"
N. 42 31 36		57048.5	t.	111189	111082	"		
N. 42 59		56979	t.	110797	111092	"	"	"
S. 33 18 30		57034	t.	111160	110908	"	"	"

SUMMARY OF ARC

Date of measurement.	Measurement made by.	Length.		Amplitude.
1762	Liesganig in Austria	172996	V. fa.	2 56 45.5
		59990	V. fa.	1 01 34.5
1762–1764	Beccaria in Italy	64899.6	t.	1 07 44.7
1763–1768	Mason and Dixon in America	538067	ft.	1 28 45
1787	Gen. Roy in Eng. and France	963918	ft.	2 38 36
1787–1802	Gen. Roy in England	1036337.0	ft.	2 50 23.38
1790–1800	Delambre and others in France	551584.72	t.	9 40 24.24
1801–1803	Svanberg in Lapland	180827.68	m	1 37 19.566
1816–1850	Struve and Tenner in Russia	1447786.783	t.	25 20 08.29
1821–1823	Plana and Carlina in Italy	126394.60	m.	1 07 31.07
1790–1791	Burrow in India	411002	ft.	1 07 59
1800–1805	Lambton and Everest in India	95721.32	ft.	1 34 56.428
1805–1825	" " "	5794598.58	ft.	15 57 39.34
1830–1847	Everest in India	2202926.196	ft.	6 03 55.97
		1961157.117	ft.	5 23 37.05
1820 abt.	Gauss in Germany	115163.725	t.	2 00 57.42
1820	Schumacher in Denmark	87436.538	t.	1 31 53.306
1830–1835	Bessel in Germany	57965.34	t.	1 30 28.98
1840–1848	Maclear in South Africa	1678373.8	ft.	4 36 48.57

MEASUREMENTS.—*Continued.*

	Value of 1 degree the earth considered as a sphere			
Middle latitude.	Length in original measure.	Length in metres.	Length of 1° based on Clarke's spheroid of 1866. M.	Measuring apparatus used.
— — — —	57077 t.	— —	— — "	Wooden Rods
— — — —	56871 t.	— —	— — "	" "
N. 44 57	57468 t.	112006	111131 "	
N. 39 12	363763 ft.	110875	111018 "	" levels
N. 50 09 27″	365040 ft.	111264	111234 "	Steel chain
N. 52 50 30	364596 ft.	111128	111275 "	" "
N. 40 11 58	57018 t.	111128	111038 "	Borda base apparatus
N. 66 20 10	111477.4 m.	111477.4	111517 "	Steel bars Struve & Tenner base apparatus Wooden rods
N. 23 18 12″	362742 ft.	110564	110748 "	Chain and bamboo rods
N. 12 32	362964 ft.	110631	110626 "	Steel chain
				" "
				Colby compensating base apparatus
N. 52 32 15	57127.2 t.	111341	111279 "	Compensating base apparatus
				" "
				" "
				Colby base apparatus

In the above table, the lengths and amplitudes of the arcs are given as deduced by the original investigators. These, as has been seen in the discussion of the earth considered as a spheroid, have been slightly changed by the different computors.

The preceding work has ended with the discussion of Clarke's Spheroid of 1866, and this considered all important arcs up to 1850. Since that time, nearly every nation has undertaken or continued its geodetic work, so that the information accrued in the last fifty years would alone demand much space. The United States has now a number of arcs, one being very important, namely, the oblique transcontinental arc stretching from the Atlantic to the Pacific along the 39th parallel. This, with the smaller arcs scattered over the country and in connection with a proposed meridian arc along the 98th meridian extending north into Canada and south into Mexico, will afford an excellent determination of the earth's figure. The work in India has been largely extended, so that at present long arcs are available in that country.

The work in Africa contemplates the extension of the meridian arc from the Cape of Good Hope to Cairo, which, if connected with the Russian arc, would give an arc of about 100 degrees in extent At the present time the French government is supervising the remeasurement of the Peru arc of 1735. The results of the remeasurement of this historic arc will be awaited with interest. What is true of these various countries in geodetical work is true of nearly every country, so that in a few years, there will be available data that ought to give an accurate determination of the earth's figure.

During the last half century, the figure of the earth has been investigated considering it an ellipsoid, of three unequal dimensions. Prominent among these investigators are Schubert in Russia and General Clarke in England. The latter's earlier determinations gave a difference of about 5400 feet between the equatorial semi-major and semi-minor axes. Later investigations by him make the difference less it being about 1500 feet. The general trend of present thought concerning the earth's figure is that it is not a three axial body. Its true shape is undoubtedly geoidal or wavy in form but the difference between its true form and a spheroid of revolution will probably be but slight. To determine the exact geoidal form will call for years of study and investigation, so that since the spheroid of revolution lends itself much more easily to computations, that form will undoubtedly be used for a number of years to come.

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Astron. Nach. for Astronomische Nachrichten.
Corr. Astron. for Correspondance Astronomique.
Mon. Corr. for Monatliche Correspondenz.
Hist. d. l. A. R. d. Sci. for Histoire de l'Academie Royale des Sciences.
L. E. D. for London, Edinburgh and Dublin Philosophical Magazine and Journal of Science; being a continuation of "Tilloch's Philosophical Magazine," Nicholson's "Journal" and Thomson's "Annals of Philosophy" (London).
Phil. Trans. Roy. Soc. Lon. for Philosophical Transactions of the Royal Society of London.

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